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TOPICAL MEETING ON OPTICAL BISTABILITY HELD AT  
ROCHESTER NEW YORK ON 15-17 JUNE 983(U) OPTICAL SOCIETY  
OF AMERICA WASHINGTON D C J W QUINN 1983

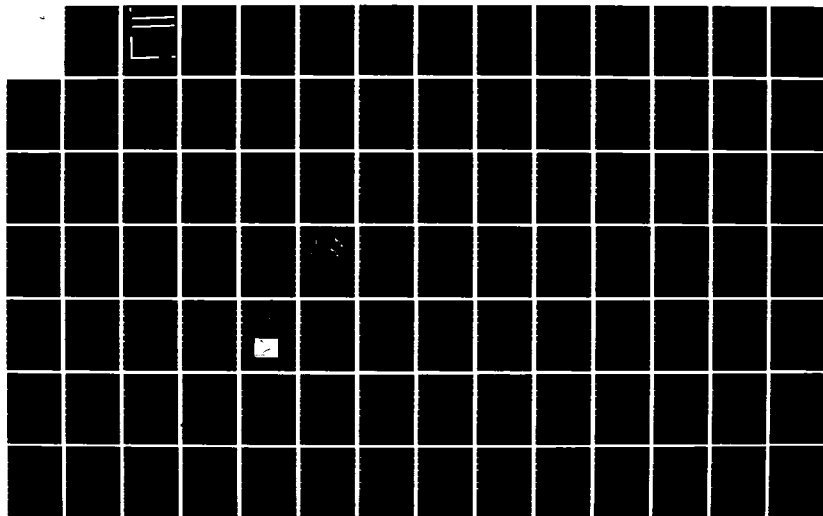
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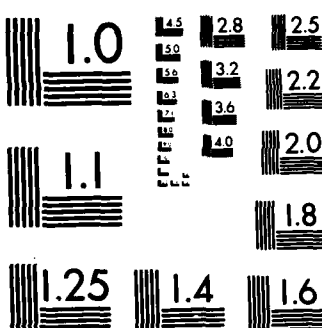
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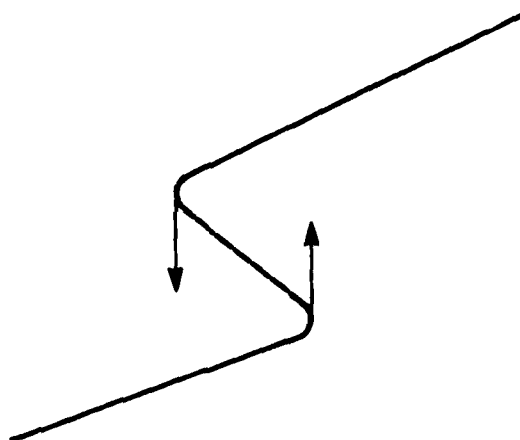
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## OPTICAL BISTABILITY



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JUNE 15-17, 1983  
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) The topical meeting on optical bistability was intended to provide an international interdisciplinary forum for the exchange of knowledge on the progress of various aspects of optical bistability and optical nonlinearities. Papers in the following areas were covered: theory, experiments, devices, material properties, instabilities and chaos and coherent switching.		



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AFOSR-83-0251

FINAL

# TOPICAL MEETING ON OPTICAL BISTABILITY

A digest of technical papers presented at the Topical Meeting on Optical Bistability,  
June 15-17, 1983, University of Rochester, Rochester, New York

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WEDNESDAY, JUNE 15, 1983

**HOYT HALL**

**8:00 AM - 10:30 AM**

**Optical Bistability I**

Hyatt M. Gibbs, *Presider*

**9:00 AM WHa1**

**Chaos and Optical Bistability: Bifurcation Structure**, K. Ikeda, Kyoto University, Japan.

The structure of bifurcations exhibited by solutions of a delay-differential equation that models a bistable optical cavity is investigated in detail.

**9:30 AM WHa2**

**Optical Bistability with Two-Level Atoms**, H. J. Kimble and A. T. Rosenberger, University of Texas at Austin, Austin, TX, and P. D. Drummond, University of Rochester, Rochester, NY.

Measurements of steady-state characteristics and of transient response are reported for optical bistability with intracavity atomic beams in both standing-wave and ring configurations.

**10:00 AM WHa3**

**Optical Bistability, Chaos in the Coherent Two-Photon Processes**, G. S. Agarwal, University of Hyderabad, Hyderabad, India, and Surendra Singh, University of Arkansas, Fayetteville, AR.

Higher-order coherent processes, such as two-photon absorption, in atomic systems in a ring cavity are shown to exhibit chaotic character following Feigenbaum scenario. The characteristics of chaotic regime and various exponents will be discussed.

**10:30-11:00 AM COFFEE BREAK**

**HOYT HALL**

**11:00 AM-12:30 PM**

**Optical Bistability II**

S. L. McCall, *Presider*

**11:00 AM WHb1**

**Turbulence and 1/f Noise in Quantum Optics**, F. T. Arecchi, Università di Firenze, Italy.

Generalized multistability and chaotic behavior are observed in several quantum optical systems. Inducing jumps between independent attractors leads to a low frequency of power spectrum of 1/f type. Experiments and theory are here presented.

**11:30 AM WHb2**

**Self-Pulsing, Breathing, and Chaos in Optical Bistability and the Laser with Injected Signal**, L. A. Lugiato, Istituto di Fisica Dell'Università, Milano, Italy, and L. M. Narducci, Drexel University, Philadelphia, PA.

In this paper we review some of the most relevant manifestations of self-pulsing in optical bistability, including the absorptive case in which self-pulsing is a multimode phenomenon.

WEDNESDAY, JUNE 15, 1983, *Continued*

**12:00 N WHb3**

**Multistability, Self-Oscillation, and Chaos in Nonlinear Optics**, H. J. Carmichael, University of Arkansas, Fayetteville, AR, and C. M. Savage and D. F. Walls, University of Waikato, Hamilton, New Zealand.

Multistability, self-oscillation, and chaos in a model for polarization switching is discussed. Chaos is also predicted in two-photon optical bistability and sub/second-harmonic generation.

**12:30 PM-2:00 PM LUNCH**

**HUTCHISON HALL**

**2:00 PM-2:10 PM**

**Introductory Remarks**

S. L. McCall, *Presider*

**2:10 PM-3:10 PM**

**Optical Chaos**

R. Bonifacio, *Presider*

**2:10 PM WC1**

**Path to Optical Chaos**, F. A. Hopf, Optical Sciences Center, University of Arizona, Tucson, AZ.

**2:40 PM WC2**

**Chaos in Dispersive Bistability**, Axel Schenzle, University of Essen, Federal Republic of Germany.

**3:10 PM-3:40 PM COFFEE BREAK**

**HUTCHISON HALL**

**3:40 PM-5:10 PM**

**Fluctuations and Instabilities**

C. M. Bowden, *Presider*

**3:40 PM WD1**

**The Physical Mechanism of Optical Instabilities**, Yaron Silberberg and Israel Bar-Joseph, Weizmann Institute of Science, Rehovot, Israel.

Instabilities in nonlinear optics are related to wave-mixing gain processes. Various self-pulsation phenomena and period doubling in optical cavities are explained and analyzed.

**4:00 PM WD2**

**Connection between Ikeda Instability and Phase Conjugation**, W. J. Firth, E. M. Wright, and E. J. D. Cummins, Heriot-Watt University, Edinburgh, U.K.

Ikeda oscillation is interpreted as a four-wave parametric instability. In a Fabry-Perot, the phase-conjugating grating induces a new dynamic instability.

**4:20 PM WD3**

**Clinical Quantum Fluctuations in Optical Bistability**, Peter D. Drummond, University of Rochester, Rochester, NY.

Absorptive optically bistable elements in the critical region have increased quantum fluctuations by a factor of  $\sqrt{n}$ , where  $n$  is the threshold photon number.

**WEDNESDAY, JUNE 15, 1983, Continued**

**4:40 PM WD4**

**Fluctuations in Optical Bistability: Experiment with Shot Noise**, S. L. McCall, Bell Laboratories, Murray Hill, NJ, and S. Ovadia, H. M. Gibbs, F. A. Hopf, and D. L. Kaplan, Optical Sciences Center, University of Arizona, Tucson, AZ.

Using a hybrid bistable optical device with a large photon shot noise, we have observed fluctuations and spontaneous transitions between the two states. A simple ladder model fits the data well.

**5:45 PM-6:45 PM DINNER (for on campus registrants only)**

**7:00 PM Buses leave from the library for the Dryden Theatre (at George Eastman House)**

**7:45 PM Film Show: "Man in the White Suit."**

**8:00 PM Buses return to the University of Rochester campus**

**9:15 PM-10:30 PM Reception and Poster Paper Review in the May Room of Wilson Commons**

**THURSDAY, JUNE 16, 1983**

**HOYT HALL**

**8:00 AM-10:10 AM**

**Bistability: Semiconductors and Novel Schemes**  
Peter W. Smith, *Presider*

**8:00 AM ThA1**

**Room Temperature Optical Bistability in Semiconductors and Realization of Optical AND Gates**, S. D. Smith, Heriot-Watt University, Edinburgh, U.K.

Recent developments in optical nonlinearities in InSb are reviewed, and new devices including an AND gate and interacting logic elements are described.

**8:30 AM ThA2**

**Advances in GaAs Bistable Optical Devices**, J. L. Jewell, S. S. Tarng, H. M. Gibbs, K. Tai, and D. A. Weinberger, Optical Sciences Center, University of Arizona, Tucson, AZ, and A. C. Gossard, S. L. McCall, A. Passner, T. N. C. Venkatesan, and W. Weigmann, Bell Laboratories, Murray Hill, NJ.

The achievement in GaAs bistable optical devices of room-temperature operation, external switch-on and switch-off, and advances in device fabrication are described.

**9:00 AM ThA3**

**Nonlinearities at the Bandgap in InAs**, Craig D. Poole and E. Garmire, Center for Laser Studies, University of Southern California, Los Angeles, CA.

Nonlinear absorption has been observed in InAs between 10° and 100°K at intensities comparable with InSb. Parameters affecting bistability will be discussed.

**9:20 AM ThA4**

**Nonlinear Optical Interfaces**, P. W. Smith and W. J. Tomlinson, Bell Laboratories, Holmdel, NJ.

We review previous theoretical and experimental work on nonlinear optical interfaces and report new experimental results that confirm the predictions of our two-dimensional Gaussian beam analysis.

**9:50 AM ThA5**

**Mirrorless Intrinsic Optical Bistability due to the Local Field Correction in the Maxwell-Bloch Formulation**, C. M. Bowden, U.S. Army Missile Command, Redstone Arsenal, AL, F. A. Hopf, Optical Sciences Center, University of Arizona, Tucson, AZ, and W. H. Louisell (deceased), University of Southern California, Los Angeles, CA.

We show that optical bistability occurs under suitable conditions for material parameters that are due entirely to the local field correction incorporated into the Maxwell-Bloch formulation.

**10:10 AM-10:30 AM COFFEE BREAK**

THURSDAY, JUNE 16, 1983, *Continued*

**MAY ROOM/WILSON COMMONS**

**10:30 AM-12:00 M**

**Poster Session I**

P. Drummond, *Presider*

**Poster ThB1**

**Optically Induced Changes in the Faraday Rotation Spectra of the Ferromagnetic Semiconductor  $\text{CdCr}_2\text{Se}_4$** , Norman A. Sanford, Sperry Research Center, Sudbury, MA.

Induced Faraday rotation in transmitted probe light that is due to a second circularly polarized pumping beam has been observed. Sense of response followed pump polarization sign.

**Poster ThB2**

**Multiparameter Universal Route to Chaos in a Fabry-Perot Resonator**, E. Abraham and W. J. Firth, Heriot-Watt University, Edinburgh, U.K.

We present analytic and computational evidence for Feigenbaum bifurcation to chaos in a folded resonator containing a Kerr medium with finite response time.

**Poster ThB3**

**Bistable Operation of Two Semiconductor Lasers in an External Cavity: Rate-Equation Analysis**, T. G. Dziura, and D. G. Hall, The Institute of Optics, University of Rochester, Rochester, NY.

The steady-state and dynamic behavior of a bistable laser resonator containing two semiconductor elements is examined theoretically.

**Poster ThB4**

**Optically-Induced Bistability in a Nematic Liquid Crystal**, Hiap Liew Ong and Robert B. Meyer, Brandeis University, Waltham, MA.

The structural bistability in an optical field of intensity  $\sim 200/\text{cm}^2$  in a nematic liquid crystal is discussed.

**Poster ThB5**

**Room-Temperature Thermal Optical Bistability in Thin-Film Interference Filters and Dye-Filled Étalons**, M. C. Rushford, D. A. Weinberger, H. M. Gibbs, C. F. Li, and N. Peyghambarian, Optical Sciences Center, University of Arizona, Tucson, AZ.

Thermal optical bistability was observed in interference filters and dye-filled étalons. CW operation and demonstration of crosstalk holds potential for studies of optical signal processing.

**Poster ThB6**

**Self-Defocusing and Optical Crosstalk in a Bistable Optical Étalon**, K. Tai, H. M. Gibbs, J. V. Moloney, S. S. Tarng, J. L. Jewell, and D. A. Weinberger, Optical Sciences Center, University of Arizona, Tucson, AZ.

Self-defocusing effects are observed in a bistable GaAs-GaAlAs superlattice étalon and compared with numerical simulations. Crosstalk via diffractive coupling is also simulated.

THURSDAY, JUNE 16, 1983, *Continued*

**Poster ThB7**

**Routes to Optical Turbulence in a Dispersive Ring Bistable Cavity Containing Saturable Nonlinearities**, J. V. Moloney, H. M. Gibbs, and F. A. Hopf, Optical Sciences Center, University of Arizona, Tucson, AZ.

Using a plane-wave analysis we show that coexistent attractors, representing distinct routes to optical turbulence, appear on a single branch of a dispersive ring bistable cavity. Some of these routes appear to arise via a tangent (saddle-node) bifurcation and are not found in the logistic map analyzed by Feigenbaum. When transverse variations in the beam profile are accounted for, a new transition sequence arises that is consistent with recent observations in a fluid dynamical experiment.

**Poster ThB8**

**Resonant Frustrated Total Reflection (FTR) Approach to Optical Bistability in Semiconductors**, Bruno Bosacchi, Western Electric - Engineering Research Center, Princeton, NJ, and Lorenzo M. Narducci, Drexel University, Philadelphia, PA.

We discuss the resonant FTR (Frustrated Total Reflection) approach to optical bistability and its relationship with alternative configurations (nonlinear, interface, waveguide, and surface-plasmon).

**Poster ThB9**

**Optical Hysteresis and Bistability in the Wave-Front Conjugation by Electro-Optic Crystals**, N. V. Kukhtarev, T. I. Semenets, and V. N. Starkov, Ukrainian Academy of Sciences, Kiev, USSR.

It is shown that, in the transmission and reflection types of the phase conjugation in the electro-optic crystals, noncavity optical bistability may be observed.

**Poster ThB10**

**Observation of Bistability in Josephson Device**, Barry Muhlfelder and Warren W. Johnson, University of Rochester, Rochester, NY.

An apparently new type of (possibly metastable) bistability has been observed in a double Josephson junction device.

**Poster ThB11**

**Theory of Multistability in Josephson Junction Devices**, Peter D. Drummond, Barry Muhlfelder, and Warren W. Johnson, University of Rochester, Rochester, NY.

Josephson junctions coupled to electromagnetic fields can have nonequilibrium phase transitions. A new theory of optical multistability in junctions coupled to waveguides is compared with simulations.

**Poster ThB12**

**Intrinsic Instabilities in Homogeneously Broadened Lasers**, Lloyd W. Hillman, Robert W. Boyd, Jerzy Krasinski, and C. R. Stroud, Jr., Institute of Optics, University of Rochester, Rochester, NY.

It is demonstrated that a single-mode homogeneously broadened laser far above threshold is unstable to the growth of additional modes.

THURSDAY, JUNE 16, 1983, *Continued*

**Poster ThB13**

**Zeeman-Coherence Effects in Absorptive Polarization Bistability**, Govind P. Agrawal, Bell Laboratories, Murray Hill, NJ.

Zeeman-coherence effects in absorptive polarization bistability are studied using a folded three-level system. The collisional decay of Zeeman coherence modifies substantially the bistable and tristable behavior.

**Poster ThB14**

**Effect of Driving Laser Fluctuations in Optical Bistability**, Charles R. Willis, Boston University, Boston, MA.

We derive and analyze the master equation in the high  $Q$  limit when the incident laser has amplitude and frequency fluctuations.

**Poster ThB15**

**Optical Bistability in Nonlinear Media: An Exact Method of Calculation**, Y. B. Band, Allied Corporation, Mt. Bethel, NJ.

Exact equations are obtained for reflection and transmission coefficients for a plane wave incident upon a slab of nonlinear material. The bistability is calculated for slabs of InSb.

**Poster ThB16**

**Self-Beating Instabilities in Bistable Devices**, J. A. Martin-Pereda and M. A. Muriel, Universidad Politécnica de Madrid, Ciudad Universitaria, Madrid, Spain.

A self-beating phenomenon for instabilities in optical bistable devices is reported. A general differential equation is presented. Liquid crystal was employed as nonlinear medium.

**Poster ThB17**

**Empirical and Analytical Study of Instabilities in Hybrid Optical Bistable Systems**, J. A. Martin-Pereda and M. A. Muriel, Universidad Politécnica de Madrid, Ciudad Universitaria, Madrid, Spain.

Conditions for instabilities in hybrid bistable devices are analyzed. Studied cases are  $I_{OUT}$  versus  $I_{IN}$  and  $\beta I_{OUT}$  versus  $VB$ . A straightforward method to achieve instabilities is presented.

**Poster ThB18**

**Distributed Feedback Bistability in Channel Waveguides**, G. I. Stegeman and C. Liao, Optical Sciences Center and Arizona Research Laboratories, University of Arizona, Tucson, AZ, and H. G. Winful, GTE Laboratories, Inc., Waltham, MA.

We analyze optical bistability in a channel waveguide using a grating for distributed feedback.

**Poster ThB19**

**Cyclotron Bistability of Electrons in Vacuum and Semiconductors**, A. E. Kaplan, Purdue University, West Lafayette, IN.

Cyclotron resonance of free (conductive) electrons in vacuum (and semiconductors) can exhibit bistable and hysteretic behavior that is due to (quasi)relativistic masseeffect of the electron.

THURSDAY, JUNE 16, 1983, *Continued*

**Poster ThB20**

**Switching Behavior of Bistable Resonators Filled with Two-Level Atoms**, E. Chang, G. Cooperman, M. Dagenais, and H. Winful, GTE Laboratories, Inc., Waltham, MA.

For material parameters of physical interest, we show that an optical bistable device cannot switch off much faster than the two-level energy relaxation time.

**Poster ThB21**

**Polarization of Bistability in Cholesteric Liquid Crystals**, Herbert G. Winful, GTE Laboratories, Inc., Waltham, MA.

The polarization state of an intense light beam in a cholesteric liquid crystal displays hysteresis and bistability as the incident intensity is varied.

**Poster ThB22**

**Simulations of the Dynamics of Optical Bistability with Fluctuations**, S. W. Koch, H. E. Schmidt, and H. Haug, Institut für Theoretische Physik der Universität Frankfurt, Frankfurt-Main, Federal Republic of Germany.

Detailed numerical solutions are presented for the dynamics of the fluctuating light field in a nonlinear resonator showing optical bistability.

**12:00 M-1:30 PM LUNCH**

**MAY ROOM/WILSON COMMONS**

**1:30 PM-3:00 PM**

**Poster Session II**

P. Drummond, *Presider*

**Poster ThB23**

**Instabilities and Chaos in TV-Optical Feedback**, G. Häusler and N. Streibl, Physikalisches Institut der Universität, Erlangen, Federal Republic of Germany.

TV-optical feedback systems are a simple realization of multi-dimensional (space/time) dynamical systems. We show some experimental results of bistable, unstable, and chaotic behavior.

**Poster ThB24**

**Polarization Switching with  $J = \frac{1}{2}$  to  $J = \frac{1}{2}$  Atoms in a Ring Cavity**, W. J. Sandle, M. W. Hamilton, and R. J. Ballagh, University of Otago, Dunedin, New Zealand.

We investigate theoretically the steady-state behavior of a ring cavity containing a gas of  $J = \frac{1}{2}$  to  $J = \frac{1}{2}$  atoms subjected to a steady magnetic field.

THURSDAY, JUNE 16, 1983, *Continued*

**Poster ThB25**

**Optical Bistability Based on Self-Focusing**, P. W. Smith and D. J. Eilenberger, Bell Laboratories, Holmdel, NJ.

We review theoretical and experimental work on self-focusing bistability and describe new experimental results in which high-order self-focusing and multistability are observed for the first time.

**Poster ThB26**

**Optical Bistability in Four-Level Nonradiative Dyes**, Zhen Fu Zhu and E. Garmire, Center for Laser Studies, University of Southern California, Los Angeles, CA.

Theory and experiments show bistability that is due to intensity-dependent nonresonant dispersion caused by an excited metastable state that is populated in the process of saturable absorption.

**Poster ThB27**

**Traveling Wave Bistability**, J. A. Goldstone and E. Garmire, Center for Laser Studies, University of Southern California, Los Angeles, CA.

We investigate bistability without feedback based on a quantum mechanical analog of the classical Duffing oscillator that leads to a bistable susceptibility tensor.

**Poster ThB28**

**Quantum Theory of Optical Excitonic Bistability**, C. W. Gardiner, University of Waikato, Hamilton, New Zealand and M. L. Steyn-Ross, York University, Toronto, Canada.

A master equation approach to the intracavity interaction of excitons, electrons, holes, and light is shown to give an explanation of optical bistability in GaAs.

**Poster ThB29**

**Optical Bistability in Semiconductors**, C. C. Sung, The University of Alabama in Huntsville, Huntsville, AL, and C. M. Bowden, U.S. Army Missile Command, Redstone Arsenal, AL.

The effect of exciton and bi-exciton excitations in semiconductors and the local field correction on the condition for optical bistability are quantitatively studied.

**Poster ThB30**

**Critical Slowing-Down in Microwave Absorptive Bistability**, A. Gozzini and F. Maccarrone, Istituto di Fisica dell'Università, Pisa, Italy, S. Barbarino, Istituto di Fisica dell'Università, Catania, Italy and I. Longo and R. Stampacchia, Istituto di Fisica Atomica e Molecolare del CNR, Pisa, Italy.

Experimental evidence and theoretical analysis of the critical slowing down in a confocal resonator tuned at the 3-3 ammonia inversion line are reported.

THURSDAY, JUNE 16, 1983, *Continued*

**Poster ThB31**

**Absorptive Optical Bistability with Laser Amplitude Fluctuations**, M. Kuś and K. Wódkiewicz, Institute of Theoretical Physics, Warsaw University, Warsaw, Poland and J. A. C. Gallas, Max-Planck-Institute of Quantum Optics, Garching, Federal Republic of Germany.

Absorptive optical bistability with fluctuations of the injected signal is investigated. In the limit of large laser linewidth a Fokker-Planck equation is derived and its stationary solution is discussed.

**Poster ThB32**

**Precise Measurements of Bistability in a CO<sub>2</sub> Laser with Intracavity Saturable Absorber**, E. Arimondo, B. M. Dinelli, and E. Menchi, Istituto di Fisica dell'Università, Pisa, Italy.

The optical bistability of a standing-wave laser containing an intracavity SF<sub>6</sub> absorber was observed versus the pumping parameter and compared with an appropriate theoretical model.

**Poster ThB33**

**Periodic Window, Period Doubling, and Chaos in a Liquid Crystal Bistable Optical Device**, Jae-Won Song, Hai-Young Lee, Sang-Yung Shin, and Young-Se Kwon, Korea Advanced Institute of Science and Technology, Seoul, Korea.

Periodic windows as well as period doublings and chaos have been observed in a twisted nematic liquid crystal bistable optical device with a delayed feedback.

**Poster ThB34**

**Effects of the Radial Variation of the Electric Field on some Instabilities in Lasers and in Optical Bistability**, L. A. Lugiato and M. Milani, Dipartimento di Fisica dell'Università, Milano, Italy.

We discuss the matter of instabilities in homogeneously broadened ring lasers and absorptive optical bistability assuming a Gaussian transverse profile for the electric field.

**Poster ThB35**

**Theory of Optical Bistability in Collinear Degenerate Four-Wave Mixing**, Richard Lytel, Electro-optics Laboratory, Lockheed Palo Alto Research Laboratory, Palo Alto, CA.

A theory of collinear degenerate four-wave mixing in isotropic media, including pump depletion, is presented and solved. Optical bistability is predicted and discussed.

**Poster ThB36**

**A Saturable Interferometer Theory of Absorptive Optical Bistability**, N. M. Lawandy and R. P. Willner, Brown University, Providence, RI.

We utilize the method of summing transmitted waves, routinely used to analyze interferometer structures, to develop an optical bistability equation that allows us to calculate switching times for both the up and down processes.

**Poster ThB37**

**Bistability in Optical Waveguides**, C. Sibilia and M. Bertolotti, Istituto di Fisica, Università di Roma, Roma, Italy.

The possibility of bistable behavior in a nonlinear transverse inhomogeneous waveguide is discussed.

THURSDAY, JUNE 16, 1983, *Continued*

**Poster ThB38**

**Nonlinear Fabry-Perot Transmission in a CdHgTe Etalon**, G. Parry, J. R. Hill, and A. Miller, Royal Signals and Radar Establishment, Malvern, England.

We extend our study of large nonlinear refractive effects in cadmium mercury telluride to investigate the transmission characteristics of an etalon.

**Poster ThB39**

**Self-Pulsing and Chaos in Inhomogeneously Broadened Single-Mode Lasers**, R. Graham and Y. Cho, Fachbereich Physik, Universität Essen GHS, Essen, W. Germany.

Self-pulsing and chaos in inhomogeneously broadened single-mode lasers is investigated in the framework of two models with 4 and 6 deg of freedom, respectively.

**Poster ThB40**

**Self-Pulsings, Hysteresis, and Chaos in a Standing-Wave Laser**, L. A. Kotomtseva, N. A. Loiko, and A. M. Samson, Institute of Physics, BSSR Academy of Sciences, Minsk, USSR.

Conditions for self-pulsing, hysteresis, and three types of chaotic behavior considering the gain and losses longitudinal structure in a standing-wave laser as a dynamic system are considered theoretically.

**Poster ThB41**

**The NMR Laser—A Nonlinear Solid-State System Showing Chaos**, E. Brun, B. Derighetti, R. Holzner, and D. Meier, Physik-Institut, Universität Zürich, Zürich, Switzerland.

We report on novel experimental observations and corresponding theoretical considerations of a system of strongly polarized nuclear spins in a solid that is placed inside the tuned coil of an NMR detector such that self-induced laser activity sets in.

**Poster ThB42**

**Optical Nonlinearity and Bistability due to the Biexciton Two-Photon Resonance in CuCl**, N. Peyghambarian, D. Sarid, and H. M. Gibbs, Optical Sciences Center, University of Arizona, Tucson, AZ.

We analyze the nonlinear dielectric function of CuCl in the vicinity of collision broadened biexciton resonance and discuss the operation of a bistable CuCl etalon.

**Poster ThB43**

**Optical Extinction Theorem Theory of Optical Multistability Bifurcations and Turbulence in the Fabry-Perot Interferometer**, R. K. Bullough, S. S. Hassan, G. P. Hildred, and R. R. Puri, UMIST, Manchester, U.K.

We connect the envelope Maxwell-Bloch equations with optical bistability (multistability) in a Fabry-Perot (FP) cavity in a rigorous and potentially quantitative way.

**Poster ThB44**

**Optical Bistability from the Dicke Model**, R. K. Bullough, S. S. Hassan, G. P. Hildred, and R. R. Puri, UMIST, Manchester, U.K.

We report on analysis of the quantum Dicke model.

THURSDAY, JUNE 16, 1983, *Continued*

**HOYT HALL**

**3:00 PM-4:30 PM**

**Enhanced Semiconductor Nonlinearities**

C. R. Stroud, Jr., *Presider*

**3:00 PM ThC1**

**Theory of Resonance Enhanced Optical Nonlinearities in Semiconductors**, H. Haug, Institut für Theoretische Physik, Universität Frankfurt, Frankfurt-Main, Federal Republic of Germany.

The optical nonlinearities of semiconductors, enhanced by one- and two-photon excitonic resonances, are treated in a unified theory. Illustrations are given for InSb, GaAs, and CuCl.

**3:30 PM ThC2**

**Room-Temperature Optical Nonlinearities in GaAs Multiple Quantum Wells**, D. A. B. Miller, D. S. Chemla, A. C. Gossard, and P. W. Smith, Bell Laboratories, Holmdel, NJ.

GaAs multiple quantum wells show large optical nonlinearities at room temperature, at wavelengths and powers compatible with laser diodes. Recent progress is summarized.

**4:00 PM ThC3**

**Nonlinear Optical Response of the Lowest Exciton Resonances in GaAs**, R. G. Ulbrich, Bell Laboratories.

**4:30 PM-4:50 PM COFFEE BREAK**

**HOYT HALL**

**4:50 PM-5:30 PM**

**Poster Open Forum; Questions and Answers**

J. H. Eberly, *Presider*



**FRIDAY, JUNE 17, 1983. Continued**

## HOYT HALL

**10:40 AM-12:20 PM**

**CDS and Thin Film Bistability**

D. Walls, *President*

**10:40 AM FB1**

**Optical Bistability and Hysteresis under Phase Conjugation in CdS Crystal.** M. S. Brodin, A. A. Borshch, V. I. Volkov, and N. V. Kukhtarev, Institute of Physics of the Academy of Sciences of the Ukrainian SSR, Kiev, USSR.

The results of theoretical and experimental investigation of the optical hysteresis in phase conjugation under degenerate six-photon mixing in CdS crystal are presented.

**11:10 AM FB2**

**Low-Power Opti-  
Cadmium Sulfide**  
Inc., Waltham, MA

Transverse optical bistability was observed with power levels of 1 mW or less. Regenerative periodic pulsations were also seen. Subnanosecond switching was expected.

**11:30 AM FB3**

**Light-Induced Nonreciprocity; Directional Optical Bistability; Spectroscopy of Induced Anisotropy; Nonlinear Sagnac Effect,** A. E. Kaplan, Purdue University, West Lafayette, IN.

Nonlinear nonreciprocity induced by the interaction of counter-propagating waves through Kerr-nonlinear index grating can cause a variety of novel directionally asymmetrical propagation effects.

**12:00 M FB4**

**Vacuum-Deposited Thin-Film Interferometers as Bistable Devices Operating Continuously at Room Temperature, S. P. Apanasevich, F. V. Karpushko, and G. V. Sinitsyn, Institute of Physics of the BSSR Academy of Sciences, Minsk, USSR.**

Bistable operation and regenerative pulsations were observed using thin-film interferometers made of different materials. Threshold input intensity is about  $400 \text{ W/cm}^2$ , switching times are  $\sim 10^{-5} \text{ sec}$ .

**12:20 PM-1:30 PM LUNCH**

We show how to perform an adiabatic elimination of fast varying variables when the atomic and field decay rates have comparable magnitudes.

**9:50 AM FA6**

**Critical Slowing Down and Magnetically Induced Self-Pulsing in a Sodium-Filled Fabry-Perot Resonator,** J. Mlynek, F. Mitschke, and W. Lange, Institut für Quantenoptik, Universität Hannover, Hannover, Federal Republic of Germany.

Transients in all-optical bistability and tristability are studied. Critical slowing down and magnetically induced self-pulsing are observed in a sodium-filled Fabry-Perot resonator.

**10:10 AM-10:40 AM COFFEE BREAK**

**FRIDAY, JUNE 17, 1983, Continued**

**HOYT HALL**

**1:30 PM-3:10 PM**

**CuCl<sub>2</sub> Liquid Crystal, and Diode Laser Bistability**

L. Narducci, *Presider*

**1:30 PM FC1**

**Multistability due to the Helical Optical Distributed Feedback**

**in the Cholesteric Liquid Crystals**, N. V. Kukhtarev, Institute of Physics of the Academy of Sciences of the Ukrainian SSR, Kiev, USSR.

Optical multistability with response time  $10^{-9}$  sec is predicted for the impurity-doped cholesteric liquid crystals.

**2:00 PM FC2**

**Bistable Injection Lasers**, Ch. Harder, K. Y. Lau, and A. Yariv, California Institute of Technology, Pasadena, CA.

Experimental results and an analysis of the switching speed (critical slowing down) and stability (induced self-pulsations) of bistable injection lasers are presented.

**2:30 PM FC3**

**Optical Bistability in CuCl<sub>2</sub>**, B. Hönerlage, J. Y. Bigot, and R. Levy, Laboratoire de Spectroscopie et D'Optique du Corps Solide, Université Louis Pasteur, Strasbourg, France.

In CuCl<sub>2</sub>, two-photon absorption to biexciton state gives rise to renormalization of the dielectric function that leads to optical bistability with fast commutation time.

**2:50 PM FC4**

**Radiation Pressure Induced Optical Bistability**, A. Dorsel and H. Walther, Sektion Physik, Universität Munich, Federal Republic of Germany, and P. Meystre and E. Vignes, Max-Planck-Institut für Quantenoptik, Federal Republic of Germany.

Optical bistability without nonlinear medium can be achieved by using the effect of radiation pressure on a mirror.

**3:10 PM-3:40 PM COFFEE BREAK**

**HOYT HALL**

**3:40 PM-4:40 PM**

**Discussion Session**

E. Garmire, *Presider*

**4:40 PM-4:50 PM CLOSING REMARKS**

Chaos and Optical Bistability: Bifurcation Structure

K. Ikeda

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The phase of the output light from a bistable optical cavity containing a nonlinear dielectric medium obeys the following differential equation with time delay:

$$dx(t)/dt = -x(t) + \pi\mu f(x(t-t_R)). \quad (1)$$

Although the equation of this class is familiar in various areas such as ecology, neurobiology, acoustics and study of electric circuit, the behavior of its solution has not been investigated in detail. In this paper we report the results of our recent numerical study of this equation. It is found that, with increase of parameter  $\mu$ , which measures the intensity of the incident light, or delay  $t_R$ , the solution of Eq.(1) exhibits transition from a stationary state to periodic and chaotic states. In the course of this transition, there appear successive bifurcations of a novel type, which form, so to call it, a hierarchy of coexisting periodic solutions: As  $\mu$  is increased, the stationary solution becomes unstable at the first bifurcation point and breaks into a number of periodic ones. These periodic solutions form a set of higher-harmonics which can coexist with each other. With further increase of  $\mu$ , each of these solutions further bifurcates into a new set of coexisting periodic solutions.

Such a bifurcation takes place successively, causing an accelerative accumulation of coexisting periodic states and making the time evolution of the solution more and more complicated, until a chaotic state sets in. In the chaotic regime, the coexisting periodic states in turn coalesce successively into fewer sets and are finally reduced to a single chaotic state with totally complicated time evolution. This type of behavior, which has never been before in any other nonlinear system, appears generic in the class of Eq.(1).

# Optical Bistability with Two-Level Atoms

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Since the first observation of optical bistability in 1976 (1), a large number of investigations of this phenomenon have appeared in the literature (2). Of particular theoretical interest has been the bistable system composed of "two-level" atoms within an optical resonator. While the theory in this case is well-advanced, only recently have experimental results been obtained (3,4). The work that we report is for absorptive bistability with an intracavity medium that approximates a collection of two-level atoms.

The intracavity medium in our experiments consists of multiple beams of atomic sodium that are optically prepumped before entering the mode volume of an interferometer. The density of the beams can be varied to provide resonant absorption  $\alpha l$  in the range 0 to 2.0. Two different collimating geometries produce Doppler-broadened absorption profiles of either 13 or 30 MHz full-width at half-maximum, as compared to the natural linewidth of 10 MHz. Our experiments have been carried out with both standing-wave and ring interferometers with values of finesse ranging from 150 to 500.

For low intracavity atomic density, our measurements of the evolution of steady-state hysteresis in absorptive bistability are in reasonable quantitative agreement with our calculations based upon a single transverse-mode theory in the mean-field limit. Absolute switching intensities for incident and transmitted fields are obtained from measured powers and empty-cavity characteristics. Values of the atomic cooperativity

parameter  $C$  are likewise computed directly without adjustment. At the largest values of atomic cooperativity investigated, we have observed marked departures from the usual steady-state description of optical bistability. An evaluation is in progress to assess whether these departures result from spatial effects (due to the large values of  $\alpha l$  or to the excitation of higher order transverse modes) or from the possible existence of dynamical instabilities in the experiment.

In an investigation of transient response in absorptive bistability, we drive our system with a near step-function change in incident intensity from zero to intensities above the upper turning point of the steady-state hysteresis cycle. As the switching increment above the turning point diminishes, the time to reach steady-state increases by more than tenfold above the empty-cavity filling time. Our results are in qualitative agreement with earlier theoretical work for plane waves; a quantitative analysis is being pursued.

Work supported by the National Science Foundation, Grant No. PHY-8211194, and by the Joint Services Electronics Program.

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Optical Bistability, Chaos in the Coherent Two Photon Processes:

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The cooperative behavior of a system of atoms contained in a ring cavity is investigated under the condition that the fundamental atomic transition is a two photon transition<sup>1</sup>. Maxwell-Block equations are used to obtain the input vs. output relation by adiabatically eliminating all the atomic variables. The fundamental equation that results is a complex two dimensional map--which in the mean field limit reduces to the standard bistability equations. The characteristics of the two dimensional map are numerically investigated. Such a map is shown to lead to chaotic behavior following the Feigenbaum<sup>2</sup> scenario. The sequence of events following first regime of chaos is a set of period halving bifurcations. The characteristics of the power spectrum in the region of chaos are presented. The effect of noise on the period doubling bifurcations in the present model, is also investigated and the connection with the theoretical predictions<sup>3</sup> is established. The changes in the dynamical characteristics of the system when atomic inversion relaxes on the same time scale as the cavity round trip time will also be discussed in detail.

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TURBULENCE AND  $1/f$  NOISE IN QUANTUM OPTICS

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Universita di Firenze, Italy

An exciting chapter of physics has been the study of fluctuations and coherence in lasers: how atoms or molecules, rather than radiating e.m. field independently, decide to "cooperate" to a single coherent field; then, for still higher excitation, how and why they organize in a complex pattern of space and time domains, with small correlations with one another (optical turbulence). Here we discuss these new features of quantum optical systems.

It is generally known <sup>1</sup> that  $n \geq 3$  degrees of freedom nonlinearly coupled may lead to multiperiodic or chaotic oscillatory behavior (turbulence). Since quantum optics, in the finite-boundary (single mode) plus semiclassical approximations, is ruled by the 5 Maxwell-Bloch equations, one expects similar behavior in quantum optical devices <sup>2</sup>. Often these instabilities are ruled out by time scale considerations. When the atomic variables have fast damping times, at any instant polarization and inversion are in quasi-equilibrium with the rather slow field amplitude, hence the evolution reduces to a one-equation dynamics (adiabatic elimination of atomic variables). That is why a gas laser beyond threshold assumes a smooth coherent behavior. But make a bad cavity, or add an external modulation as done for Q-switching or mode-locking: then one easily gets a three-variable dynamics, sufficient to yield chaos, for particular values of the coupling constants. What was initially considered as a "bad" or "dirty" behavior (self-pulsing, irregular mode-locking) is nowadays studied as a relevant phenomenon. Furthermore, when many domains of attraction coexist (optical multistability) and an external noise allows for jumps among them, a low frequency power spectrum appears with a shape  $f^{-\alpha}$  ( $\alpha \sim 1$ ) like the  $1/f$  noises familiar in many systems <sup>3, 4</sup>.

Equivalent to the three Lorenz first order eqs. <sup>1</sup> is a system of two 1st order eqs. (or one 2nd order eq.) plus an external modulation. An

example is the driven Duffing oscillator

$$\ddot{x} + \gamma \dot{x} + \omega_0^2 x - \beta x^3 = A \cos \omega t \quad (1)$$

which can be experimentally realized with an electronic circuit <sup>5, 3</sup>.

The potential corresponds to a single minimum. For different control parameters  $\mu$  (either modulation amplitude  $A$  or frequency  $\omega$ ) it may give a sequence of subharmonic bifurcations leading eventually to chaos.

Noise is not essential (deterministic chaos), but if we add it, the number of subharmonic bifurcations before chaos becomes smaller and smaller. This can be put in terms of a scaling law where the variance of the external noise appears somewhat as a modification of the control parameter <sup>5</sup>.

Let us now change the sign of the potential, getting two stable valleys. Depending on the initial conditions, we have two independent attractors. Increase  $\mu$  until they both get strange. Now, addition of a random noise may trigger jumps from one to the other. These jumps give a low frequency divergence in the power spectrum <sup>3</sup>. Here, random noise is essential to couple the two strange attractors otherwise independent. After the first evidence of the jumping phenomenon, a similar effect was observed in a Q-modulated CO<sub>2</sub> laser <sup>4</sup>. It corresponds to a set of 2 coupled rate eqs., with time dependent cavity losses  $k(t)$ , that is, calling  $\Delta$  the population inversion and  $n$  the photon number, to

$$\begin{aligned} \dot{\Delta} &= R - 2 G n \Delta - \gamma_{11} \Delta \\ \dot{n} &= G n \Delta - k(t) n \end{aligned} \quad (2)$$

where  $k(t) = k_0 (1 + m \cos \omega t)$ .

Fig. 1a shows generalized bistability, that is, the simultaneous co-existence of two attractors in the phase space  $(\dot{n}, n)$ . Increasing the modulation depth  $m$ , the attractors become strange and the power spectrum displays a low frequency divergence (fig. 1b).

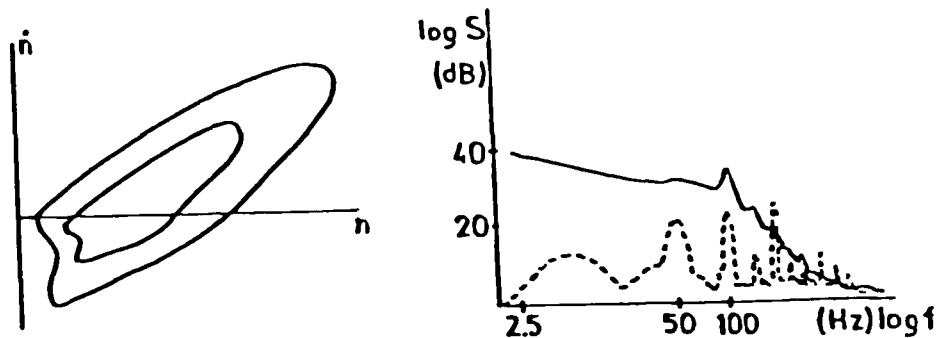


Fig. 1

Bistability and  $1/f$  noise in a  $\text{CO}_2$  laser with loss modulation

a) coexistence of two attractors (period 3 and 4 respectively). The two superposed spectra correspond to two starts with different initial conditions

b) comparison between the low frequency cut-off (dashed line) when the two attractors are stable and the low frequency divergence (solid line, slope  $\alpha = 0.6$ ) when the two attractors are strange

Experiments <sup>3, 4</sup> show that nonlinear driven systems yield power spectra with a low frequency divergence  $f^{-\alpha}$ ,  $\alpha$  being around 1 whenever the following conditions are fulfilled: i) the system is multistable, that is, it has 2 or more attractors; ii) the attractors are near to be destabilized or they have just become unstable; and iii) the system is "open" to external fluctuations, i.e., the presence of noise is essential to yield jumps between different basins of attraction. The above conditions show that we are in presence of a phenomenon which occurs beyond the usual approach to chaos by either one of the current scenarios. A simple jump between two attractors is not sufficient to explain the phenomenology. Indeed, experiments <sup>3, 4</sup> show that, when leaving an attractor, the representative point in phase space

has a long erratic motion before landing onto another attractor. This "transient" regime is made of motions among repulsive orbits.

A dynamics in terms of a recursive map must allow for at least two independent attractors. The simplest one-dimensional map with two attractors must have two extrema<sup>7</sup>. Hence we study a cubic map in the interval  $(-1, 1)$

$$x_{n+1} = (a-1)x_n - ax_n^3. \quad (3)$$

To account for item iii) the map will be disturbed by white noise with r.m.s. between  $10^{-7}$  and  $10^{-5}$ . Up to a value  $a = \bar{a} = 3 \cdot 3 / 2 + 1 = 3.598\ 076 \dots$ , the motion is confined either on the interval  $(-1,0)$  or  $(0,1)$  with qualitative features alike the well known logistic map. For  $a = \bar{a}$ , we may still have two independent attractors, whose domains, however, are interlaced in complicated ways over the interval  $(-1,1)$ .

The simplest stable pair of attractors A,B above  $\bar{a}$  is a pair of period-3 attractors which are superstable for  $a_s = 3.981\ 797 \dots$ . These period-3 attractors disappear for  $a = \tilde{a} = 3.982\ 000\ 642 \dots$ . For  $a_s < a < \tilde{a}$  (fig. 2)

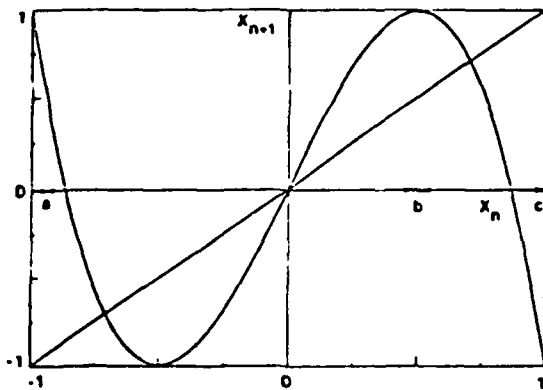


Fig.2 - Cubic map for  $\tilde{a}$ . a,b, and c show (not in scale) the intervals covered by one of the two period-3 attractors.

the presence of a small amount of noise makes it easy to leave one attractor and jump toward the other one. Before landing into the other attractor, the representative point wanders on the available space through a long transient, because of the complex structure of the two basins of attraction. The corre-

sponding low frequency power spectra, for different noise levels, are given in fig. 3.

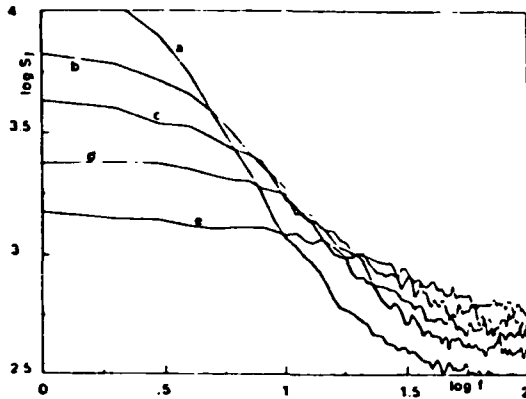


Fig.3 - Power spectra for  $a=\tilde{a}$ , and for increasing noise levels  $\sigma$ , that is, a)  $5 \times 10^{-7}$ , b)  $10^{-6}$ , c)  $2 \times 10^{-6}$ , d)  $4 \times 10^{-6}$ , e)  $10^{-5}$ . They can be fitted by  $f^{-\alpha}$ , with  $\alpha$  decreasing from 1.5 for a) to 0.5 for e)

These spectra show a power law region extending over about one decade with a slope between 0 and 2. They appear qualitatively in agreement with the experimental spectra of Refs. 3 and 4.

A model explanation of the above spectra was given <sup>7</sup> in terms of jump processes among three regions of phase space (the two attractors and the intermediate transient), by taking the jump probabilities decorrelated from the internal motions within the three regions. This leads to a power spectrum made of three Lorentzians which fit well the spectra shown in fig. 3.

Extrapolating the model to a many attractors situation we formulate the conjecture that for the limit of a highly multistable system (large number of attractors), the low frequency part  $f^{-\alpha}$  has the exponent  $\alpha = 1$ . This is akin to the well known model for  $1/f$  noise in equilibrium systems <sup>8</sup>, where the  $1/f$  slope is the sum over a large number of Lorentzian curves suitably weighted.

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SELF-PULSING, BREATHING AND CHAOS IN OPTICAL BISTABILITY AND THE LASER  
WITH INJECTED SIGNAL

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The subject of spontaneous pulsations in Optical Bistability has stimulated considerable interest following its prediction by Bonifacio and Lugiato<sup>1</sup> in the framework of the plane-wave, ring cavity model and the realization by McCall<sup>2</sup> of an electro-optical converter of cw coherent light into pulsed radiation. In a subsequent important development Ikeda<sup>3</sup> showed that, in the dispersive case, the Bonifacio-Lugiato instability leads to chaotic behavior (optical turbulence). This was later observed in a hybrid device by Gibbs, et al<sup>4</sup>.

In this paper, we review some of the most relevant manifestations of self-pulsing in Optical Bistability. First, we consider the absorptive case in which self-pulsing is a multi-mode phenomenon because the resonant mode remains stable while some of the sidebands can develop instability<sup>5</sup>. On the strength of the "dressed mode formalism" of optical bistability<sup>6</sup>, which represents an extension of Haken's theory of generalized Ginzburg-Landau equations for phase-transition like phenomena in systems far from equilibrium<sup>7</sup>, we have arrived at an essentially analytical understanding of this phenomenon. When only

the nearest two sidebands of the resonant mode become unstable, we have identified the entire domain of existence of the self-pulsing state in the plane of the control parameters. This region is divided into a soft- and a hard-excitation domain, where self-pulsing develops with a behavior that is reminiscent of second- or first-order phase transitions, respectively. In the hard-excitation region, one self-pulsing and two steady-state solutions are found to coexist, yielding hysteresis cycles that involve self-pulsing states along one of the branches. On approaching the boundary between the self-pulsing and the precipitation domains, the self-pulsing state becomes unstable. The instability is evidenced by a marked "breathing" behavior of the pulse envelope, whose transient nature has been linked to the unstable character of the limit cycle that bifurcates from the self-pulsing solution at the instability threshold.

When dispersive effects become important, self-pulsing behavior can arise even in the single-mode regime described by the well known mean field model<sup>8,9</sup>. For appropriate values of the control parameters, a large segment of the high transmission branch becomes unstable leading to periodic and irregular (chaotic) self-pulsing. On approaching the chaotic domain from either boundary, one finds a sequence of period doubling bifurcations of the type described by Grossman and Thomae<sup>10</sup> and by Feigenbaum<sup>11</sup> in the framework of the theory of discrete maps. As true also with most other dynamical models, the chaotic domain of this single-mode dispersive bistability contains "windows" of periodicity.



We have characterized the behavior of bifurcating solutions and their critical slowing down in the neighborhood of each threshold with asymptotic power laws. These will be reviewed in detail. In the purely dispersive limit, we also show that our model reduces to the one analyzed recently by Ikeda and Akimoto<sup>12</sup>. If the absorbing medium is converted into an amplifier, the optically bistable system becomes a laser with an injected signal. We assume that the laser is above threshold and that the incident field frequency is detuned from the operating frequency of the laser. Obviously, when the injected field has a vanishingly small amplitude, the laser oscillates at its own frequency. On the other hand, when the incident field intensity is large enough, the entire system is expected to oscillate with the incident frequency (this phenomenon is well-known as injection locking). We have analyzed the behavior of this system over the entire range of variation of the incident intensity below the locking threshold<sup>13</sup>. For a small incident field, the output power exhibits oscillations with a frequency equal to the mismatch between the incident and the laser frequencies, as originally proposed by Spencer and Lamb<sup>14</sup>. On increasing the incident intensity, the output becomes chaotic. A further increase in the injected signal leads to an inverted sequence of period doubling bifurcations with one- and two-periodic solutions exhibiting undamped breathing over limited domains. Finally, in the vicinity of the injection locking threshold, the system develops a chaotic spiking regime of an entirely different character from the irregular oscillations noted above. From our results, it is reasonable to speculate that the observation of chaos should be accessible with an all-optical system.

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Multistability, self-Oscillation and Chaos  
in Nonlinear Optics

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Summary

We describe a system comprising two circularly polarized ring-cavity modes coupled via a  $J=\frac{1}{2}$  to  $J=\frac{1}{2}$  transition. Both modes are equally excited by a linearly polarized incident laser. The steady-state transmission of the cavity shows optical tristability and polarization switching as predicted by Kitano et al.<sup>(1)</sup> and recently observed.<sup>(2)</sup> We generalise the model of Kitano et al. with the inclusion of saturation and absorption. In addition to optical tristability we find quadrastability (four stable output configurations for the same input intensity), self-oscillation, and chaos.<sup>(3)</sup> We have studied the chaotic region numerically as the incident laser intensity and the detuning of the cavity modes is varied. The routes to chaos appear to be analogous to those in the Lorenz equations. For a

decreasing incident intensity the passage to chaos involves a new period-doubling sequence of periodic windows embedded in chaos, where in each periodic window conventional period-doubling of the Feigenbaum type is observed. We have estimated the experimental parameters required to observe chaos in this system and find that power requirements are modest, although the limitations imposed by the adiabatic elimination and single-mode assumptions used in our theory cause some difficulties -- chaos may well remain, however, when these theoretical restrictions are relaxed.

We also predict chaos in two similar systems where a pair of ring-cavity modes are coupled via a two-photon transition and a second-order nonlinear susceptibility, respectively. Two-photon optical bistability has been discussed by various authors, where it has been the practise to neglect the Stark shift associated with the levels mediating the two-photon transition. With the inclusion of Stark shifts self-oscillations and chaos have been found in a model for two-photon bistability. For second-harmonic generation in a ring cavity McNeil et al. predicted the possibility of self-oscillation some years ago.<sup>(4)</sup> We report that with the inclusion of a cavity detuning, omitted from this earlier work, this system may also show chaos.

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# NOTES

## **Chaos in Optical Bistability**

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### **Abstract:**

Experiments on chaos in a hybrid optically bistable system with delayed feedback are presented. Special attention is paid to the connection of these experiments to the issue of what chaos is, and why it has recieved such considerable attention in recent years.

### Summary

The phenomenon of chaos has been a topic of considerable study in the last few years. Chaos is defined qualitatively as the appearance of apparently-random behavior in systems that are deterministic. A familiar example of such a case is a pinball machine. The recent interest in chaos has been spurred by the hypothesis that chaotic behavior can be defined objectively; it results from deterministic dynamics in which all motions are unstable (by the usual criteria of exponential growth of a small perturbation). Experimental evidence that random motions are due to such instability is still difficult to come by. Instead attention is focused on the periodic motions associated with chaos (called "the path to chaos"), since they are more amenable to theoretical prediction. A widespread interest in the "period doubling sequence" (the periods of different waveforms are related to each other by factors of two) has been spurred by the Feigenbaum universality hypothesis<sup>1</sup> which gives specific predictions that can be tested experimentally.

Ikeda<sup>2,3</sup> was the first to predict that chaos could be observed in optical bistability, and also suggested the use of a hybrid bistable system of the sort that we have used.<sup>4,5</sup> The present device uses an electro-optic modulator as the nonlinear element, with a delay in the feedback provided by a 1.1 km multimode optical fiber. This gives a response time  $\approx 1\mu\text{s}$  and a delay time of  $6\mu\text{s}$ . With such a fast system, we can take spectra of the output in real time, making the identification of the output waveforms straightforward. We find that waveforms converge rapidly to their asymptotic form. Most importantly, in our experiment, the output follows a change in the device parameters adiabatically, which is difficult to



accomplish in experiments with much slower response times.

The hybrid device normally followed a period doubling route to chaos, and the quantitative response of the system is consistent with theory. We find that the period doubling sequence cannot be explored in detail, since device noise limits the number of waveforms.<sup>6</sup> This limitation is itself in qualitative accord with theory, but the quantitative aspects of the response to noise is in substantial disagreement with theory. In addition we find that the path to chaos is typically nonunique in our device. We observe waveforms other than those of the period doubling sequence.<sup>7</sup> These observations are associated with hysteresis loops, which we can trace out in our device due to the fact that the waveform tracks the device parameters adiabatically. In general, our results indicate that the study of the path to chaos is not as decisive a test of theory as was originally hoped.

In addition to observations in hybrid devices, we have found all optical systems which respond in erratic ways. While the response of these devices is similar to the hybrid, we have not yet seen the period doubling sequence, which makes it difficult to claim that the device response is chaotic.

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Chaos in Dispersive Bistability

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# NOTES

## The Physical Mechanism of Optical Instabilities

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Nonlinear optical systems can transform a continuous wave input beam, if it is intense enough, into an oscillating light output. The system which has been most thoroughly studied is an optical resonator, usually of the ring type, containing either a Kerr medium<sup>1,2</sup> or a two-level medium.<sup>3</sup> Unfortunately, the mathematical approaches of these studies do not lend themselves to a tractable intuitive explanation of the self-pulsation phenomena, and do not identify the physical origin of instabilities. In this paper we consider a different approach, showing that optical instabilities stem from one source, namely, gain generated by four-wave mixing processes. This gain, together with feedback supplied by the resonator, leads to oscillation of sidebands of the input field. The beating between these sidebands and the input field causes the oscillating output intensity. The salient feature of our approach lies in its intuitive appeal, namely, it is possible to predict the behavior of the system for a wide range of parameters. Moreover, our model enables understanding of the system's evolution beyond the first bifurcation, something usually not possible with the other approaches.

First, consider the interaction of a strong wave  
 $E_0 = A_0 \exp[i(\omega_0 t - k_0 z)] + \text{c.c.}$  with two symmetrical sidebands

$E_1 = A_1 \exp\{i[(\omega_0 - \Omega)t - k_1 z]\} + \text{c.c.}$  and  $E_2 = A_2 \exp\{i[(\omega_0 + \Omega)t - k_2 z]\} + \text{c.c.}$ , in a sluggish Kerr medium having a time constant  $\tau$ . The equations governing the changes of  $E_0$  and  $E_1$  are:

$$\frac{dA_1}{d\zeta} = -i A_0 A_0^* A_1 - i \frac{1}{1-i\Omega\tau} A_0 A_0^* A_1 - i \frac{1}{1-i\Omega\tau} A_0 A_0 A_2^*$$

$$\frac{dA_2}{d\zeta} = -i A_0 A_0^* A_2 - i \frac{1}{1+i\Omega\tau} A_0 A_0^* A_2 - i \frac{1}{1+i\Omega\tau} A_0 A_0 A_1^*$$

where  $\zeta = k_0 n_2 z$ , and we assume  $|A_0| \gg |A_1|, |A_2|$ . There are two processes, described by the second and third terms on the rhs of the equations, which may contribute gain. The first one, which we denote as Rayleigh gain, is due to the interaction of each sideband with the main input field, and it results in gain to the lower frequency and in loss to the upper one. These gain and loss are maximized at  $\Omega = \tau^{-1}$ . The other process is four-wave mixing which may amplify both sidebands. When the nonlinear medium is enclosed in a cavity, oscillation at these sidebands may start from noise. Oscillation threshold and frequencies were calculated from the requirements of unit gain and the resonance condition. These calculation are simplified for limiting cases in which one of the processes is dominant. The following results were derived for a cavity with high mirror reflectivity  $R$  and transit time  $Tr$ :

1. For  $Tr \gg \tau$ , and a tuned or slightly detuned cavity, several cavity modes, shifted from  $\omega_0$  by approximately  $\tau^{-1}$ , oscillate through Rayleigh gain. The output is a complex beat pattern, as shown in figure 1a. The threshold for this instability is  $n_2 k_0 |A_0|^2 = 3(1-R)$  for a tuned cavity. This type of self-oscillation was not predicted before.

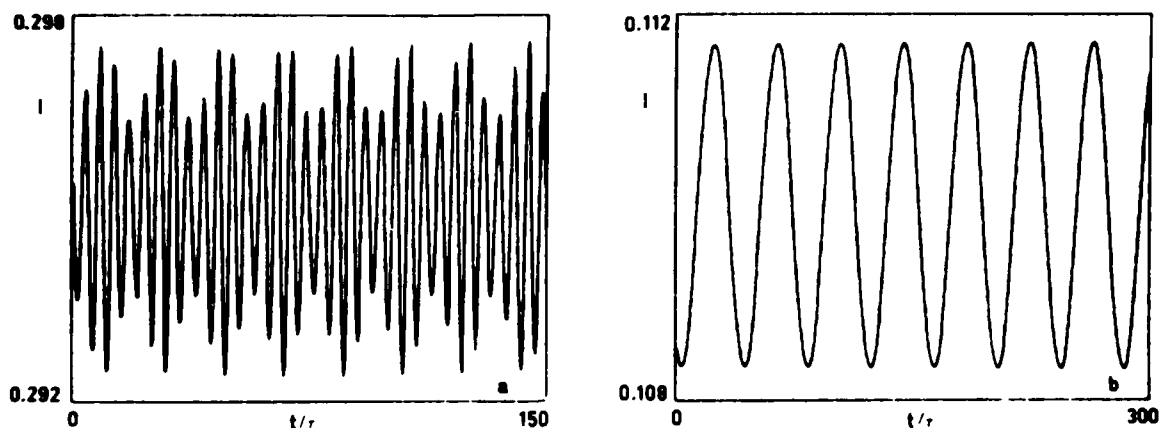


Figure 1. Light intensity vs. time in a ring cavity with  $Tr/\tau=20$ . In a) the system is tuned, and the oscillation has a frequency of  $\tau^{-1}$ . In b) the detuning is 2.9, and the oscillation has a period of  $2 Tr$ . In both cases the power is slightly above the threshold value, so that the depth of modulation is small.

2. When the detuning is close to  $\pi$ , but still below  $\pi - \sqrt{3}(1-R)$ , oscillations due to four-wave mixing occur with a period of  $2Tr$ .
3. If the cavity is in the bistable regime, i.e. detuning below  $-\sqrt{3}(1-R)$ , the oscillations occur with a period of  $Tr$ , again through four-wave mixing. Only the upper branch is unstable.<sup>1</sup>
4. For  $\tau \gg Tr$  and a tuned cavity, the system oscillates only if  $\tau(1-R) < Tr$ . The oscillation is dependent on the system parameters, and can have any value.<sup>2</sup>

Harmonics of an oscillating sideband are generated by the wave-mixing process  $A_0^* A_1 A_1$ . Since this is a source term, harmonics are always present. What

about period doubling and other bifurcations? Assume two frequencies,  $\omega_0$  and  $\omega_1$ , are present, thus beating at  $\Omega$ . A third field at  $\omega_3 = (\omega_0 + \omega_1)/2$  has gain through the interaction  $A_0 A_1 A_3^*$ . (Note the difference between gain and source terms). Whether this field oscillates will be determined by the cavity resonance condition. If it does - it beats at  $\Omega/2$ , i.e. period doubling. This process, repeating itself, is the basic mechanism of successive period doubling in a nonlinear optical system.

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Connection between Ikeda instability and phase conjugation

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Period-doubling to chaos (1) and phase conjugation are two nonlinear optical phenomena which have attracted enormous but largely independent interest in recent years. In this paper we establish a close connection between the two phenomena, both being forms of near-degenerate four-wave mixing.

If we consider a strong pump wave  $E$  at frequency  $\omega$ , and a weak signal wave  $\epsilon$  at a nearby frequency  $\omega + \Omega$ , in a medium with third order nonlinearity  $\chi$ , supposed real and nondispersive, the dominant nonlinear polarization source terms are

$$\chi |E|^2 E e^{-i\omega t}; \quad \chi |E|^2 \epsilon e^{-i(\omega+\Omega)t}; \quad \chi E^2 \epsilon^* e^{-i(\omega-\Omega)t}$$

The first term describes nonlinear refraction, and thus, in a resonator, optical bistability. The last, especially for  $\Omega = 0$ , is identifiable as a phase conjugation term: more generally, as four-wave mixing. The second describes a change in refractive-index at  $\omega + \Omega$  induced by  $E$ : in particular it will cause the resonant frequencies of a cavity to tune with pump intensity: we term this process "transphasing". Figure 1 shows this effect as a lateral movement of the gain spectrum with increasing pump intensity.

Suppose now one feeds back the signal and idler in a ring resonator: on each round trip the signal and idler experience an attenuation  $B$  and a phase shift  $\phi_0$ . The threshold condition takes the form

$$e^{-i\Omega t_R} \begin{pmatrix} \epsilon \\ \mu^* \end{pmatrix}_{t+t_R} = \begin{pmatrix} Z(1+i|E|^2) & iZE^2 \\ -iZ^*E^{*2} & Z^*(1-i|E|^2) \end{pmatrix} \begin{pmatrix} \epsilon \\ \mu^* \end{pmatrix}_t \quad (1)$$

where  $t_R$  is the cavity round trip time and  $Z = B \exp(i(\phi_0 + |E|^2))$ . This condition is exactly that governing Ikeda instability (1,2), if we interpret  $E$  as the internal field. Figure 1 shows the gain of an external probe field (i.e. (1) with a source term) versus  $\Omega$  and  $|E|^2$ . Clearly the maximum gain occurs whenever transphasing allows both signal and idler to be cavity resonant. If  $\omega$  is also resonant, this coincides with bistability, but if  $\epsilon$  and  $\mu$  are adjacent cavity modes, then  $2\Omega = 2\pi/t_R$ , so that four-wave oscillation gives a  $2t_R$  modulation to the total field - the Ikeda instability!

There are an infinity of degenerate solutions  $\Omega = (2n+1)\pi/t_R$ , corresponding to more widely separated modes: a finite response time (or phase mismatch) will obviously impose a finite bandwidth on the gain spectrum in Fig. 1, thereby raising this degeneracy: c.f. (3).

Higher bifurcations can be given a physical interpretation along similar lines: the dispersion relation around  $\omega$  is so strongly modified by large amplitude  $2t_R, 4t_R \dots$  oscillations that each mode can support several different frequencies with the same mean wavelength: this explains the separate bifurcation sequences seen in (3) with finite  $\tau$ .

If we now consider a folded (Fabry-Perot) resonator, we expect the same general picture, but with differences due to the fact that each of  $E, \epsilon$  and  $\mu$  now comprise two counter-propagating components, giving rise in particular to phase-conjugate reflections. Figure 2 shows that the probe spectrum for such a cavity is indeed considerably more complex than Fig. 1. We draw particular attention to the pair of prominent peaks in the

wings with no corresponding central peak. These peaks can be shown to arise from the phase grating due to interference of the counterpropagating waves: if this grating is washed out by diffusion, these peaks vanish (4): they are thus a consequence of nonlinear nonreciprocity.

Given the above physical considerations it is not surprising that these instabilities survive when transverse effects are included: Fig. 3 shows this for a Gaussian pump beam in a Fabry-Perot cavity. There is a band of  $t_R$ -oscillation due to nonlinear nonreciprocity, the first indication of a dynamic effect of this kind. Physically, the nonlinear medium (a thin central slice in this case) is acting as an intra-cavity phase conjugate mirror.

A close physical connection thus exists between Ikeda instabilities and four-wave mixing, especially phase conjugation. One important facet of this connection is that a full quantum optical treatment exists for phase conjugation, and predicts that "squeezing" can occur (5): adaptation of these results to nonlinear resonator will be of great significance.

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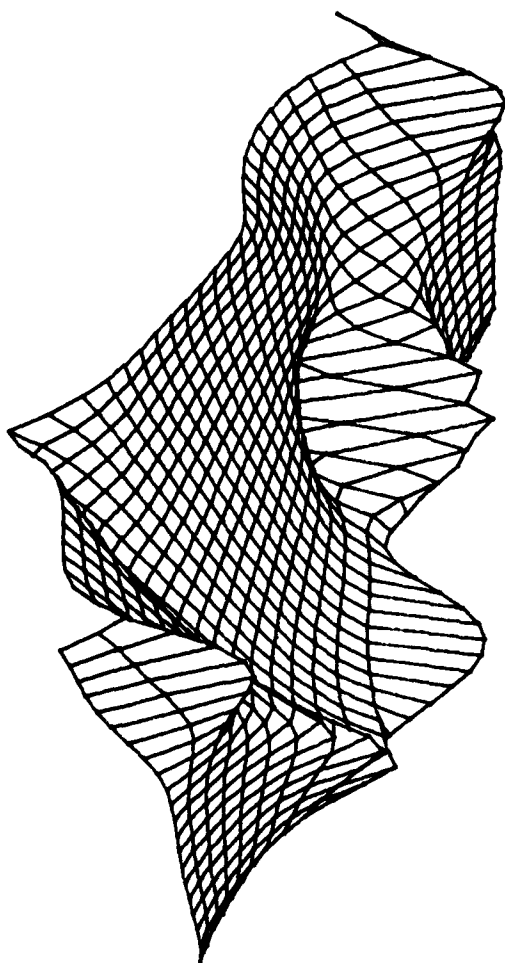


FIG. 2

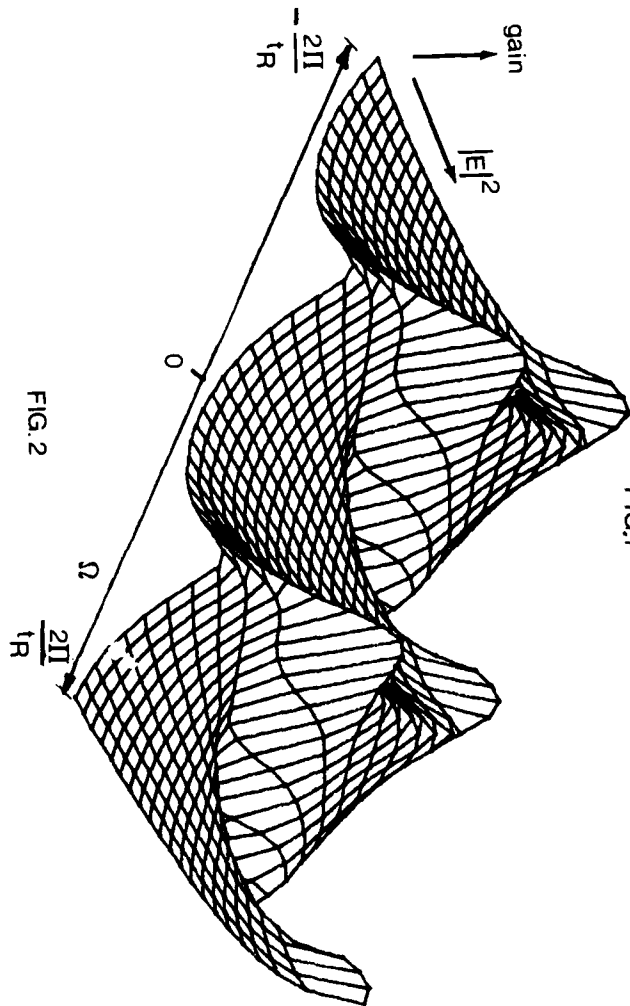
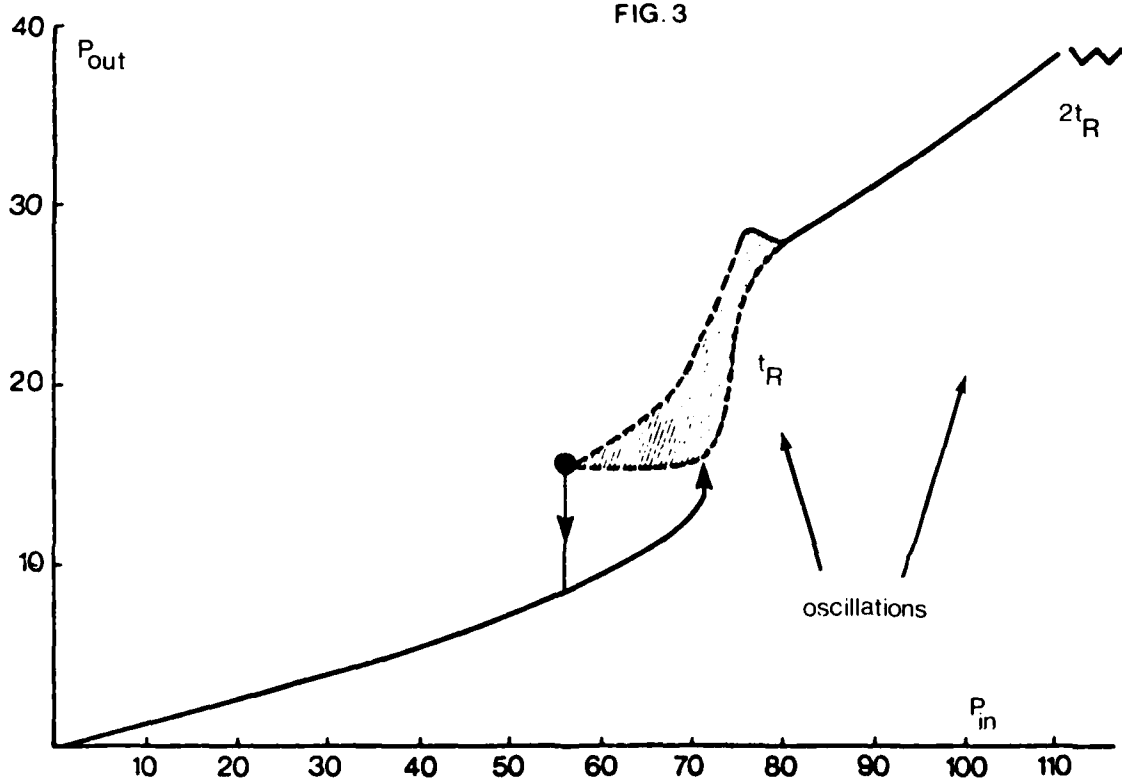


FIG. 1

FIG. 3



## Critical Quantum Fluctuations in Optical Bistability

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Intrinsic optical bistable elements that have been investigated to date typically are either high-Q devices dominated by the interferometer relaxation, or else are fast switching devices dominated by relatively slow population transfers. In the case of the one-mode quantum theory of optical bistability with two-level atoms, these limits can be treated in a unified way with a single Fokker-Planck equation.

In the critical region, a great increase in fluctuations occurs. This has the potential of allowing quantum fluctuations to be directly observed, either through fast quantum tunnelling measurements or through observations of intensity correlations. However the standard calculation of intensity correlations, which involves linearizing the Fokker-Planck equation, cannot be used to calculate critical fluctuations.

A technique of treating the critical region is therefore presented in which the Fokker-Planck equation is expanded to cubic order in the relative fluctuations. It is found that a useful simplification occurs in the limits of high-Q or of population-dominated optical bistability respectively. In the first case, the Fokker-Planck equation depends only on the interferometer mode intensity; in the second case it depends only on the population inversion.

In each case, large increases in fluctuations occur at the critical point. The resulting equations have the structure of Ginzburg-Landau equations often found in equilibrium critical-point phase transitions. This allows the calculation of the intensity correlations in a straightforward way. In the case of the high-Q interferometer, for example, the intensity fluctuations increase by a factor of  $\sqrt{n}$ , where  $n$  is the threshold photon number.

An unexpected result is that in each case, with purely absorptive optical bistability, identical equations result in either the P-representation or the Wigner representation. This suggests that near an absorptive critical point the fluctuations are large enough so that a quasi-classical description of the field is sufficient.

## Fluctuations in Optical Bistability: Experiment with Shot Noise

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The behavior of an optical bistable system in which the fluctuations play the dominant role has been studied on the basis of a Fokker-Planck description by Schenzle and Brand.<sup>1</sup> However, all the analyses so far approximate the fluctuations as arising from a constant external noise source described by a  $\delta$ -correlated, Gaussian random process.<sup>2,3</sup> In this paper intensity-dependent shot noise fluctuations of a bistable system are experimentally studied for the first time, using a hybrid bistable optical device, and analyzed by a simple "ladder" model that fits the data well.

In the present experiment a hybrid device consisting of an electro-optic crystal between crossed polarizers is employed as shown in Fig. 1. The fluctuations are introduced and controlled by attenuating the transmitted light as it passes through the neutral density filter, ND. The photomultiplier (PM) is operated in a single-photon-counting mode in which one standard pulse is generated for each PM pulse within the discriminator window. The percentage noise is larger but the absolute magnitude of the fluctuations is smaller when the system is in the lower state rather than the upper state. Data have been taken in two modes. In the first, the bistability operating point is selected so that switching back and forth occurs, and both tunneling times and the distribution of the transmitted voltages (Fig. 2) are recorded. In the other mode, the operating point can be anywhere within the bistable loop; the system is forced into one state by an external pulse and the return time to the original state is recorded (Fig. 3).

In the "ladder" model we assume a collection of two-level atoms in which only a few are excited. The decay rate is  $1/\tau$  due to spontaneous emission, while the rate for photon absorption and excited atom creation is  $R_N/\tau$ . If  $P_N$  is the probability that the bistable device has exactly  $N$  excited atoms, then  $P_N$  evolves according to the master equation:

$$\frac{dP_N}{d(t/\tau)} = -NP_N + (N+1)P_{N+1} - R_N P_N + R_{N-1} P_{N-1} \quad (1)$$

where

$$R_N \equiv \tau C_0 T(V_N) = \frac{1}{2} \tau C_0 [1 - 0.965018 \cos(GV_N)] \quad (2)$$

where  $C_0$  is the counting rate for maximum transmission and  $T(V_N)$  is the transmission with voltage  $V_N$  from the PM,  $GV_N$  is the voltage across the modulator, and  $G$  is the feedback gain.

One can obtain the non-steady state solution of Eq.(1) by assuming  $P_N \equiv e^{-\lambda t/\tau}$  and calculating the eigenvalue  $\lambda$  for a given counting rate  $C_0$ , amplifier bandwidth  $1/\pi\tau$ , and gain  $G$ . It can be shown, using the equations of motion for the probability that the device is in the lower or upper state, that the tunneling times are related to the eigenvalue as:  $\lambda = (1/\tau \uparrow + 1/\tau \downarrow)\tau$ . Thus the model allows us to obtain the return times after forced switching vs the gain and vs. the amplifier upper-roll-off frequency. Within the experimental errors, the simulated curves agree well with the experimental data (Figs. 3, 4). Comparisons with Fokker-Planck results are under way.

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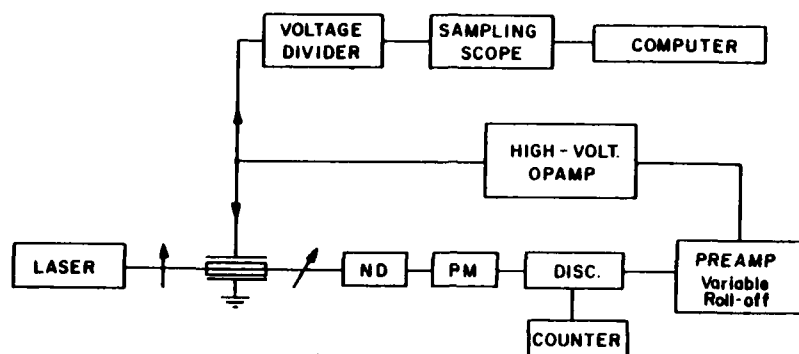


Figure 1. The experimental apparatus.

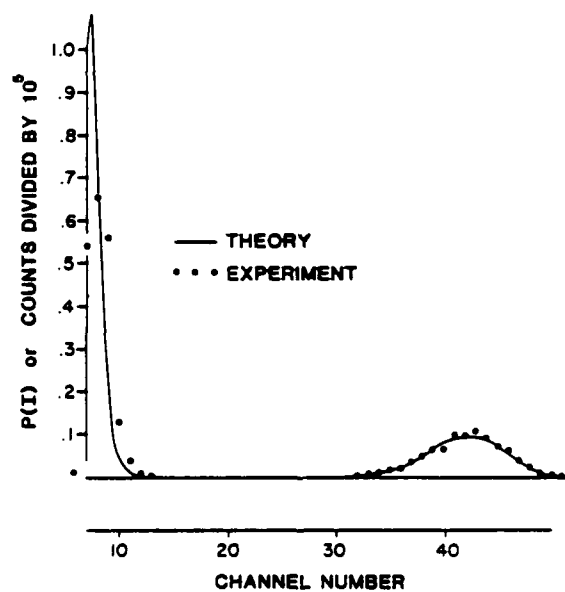


Figure 2. The open circles are the histogram of output voltage occurrences. Zero voltage is about channel 6, the lower state is about channel 8, the upper state about channel 42. The computer-simulated histogram is shown as the solid curve.

Figure 3. Return time after forced switching to the lower state ( $\uparrow$  points) and to the upper state ( $\downarrow$  points) as a function of feedback gain setting (in arbitrary units).

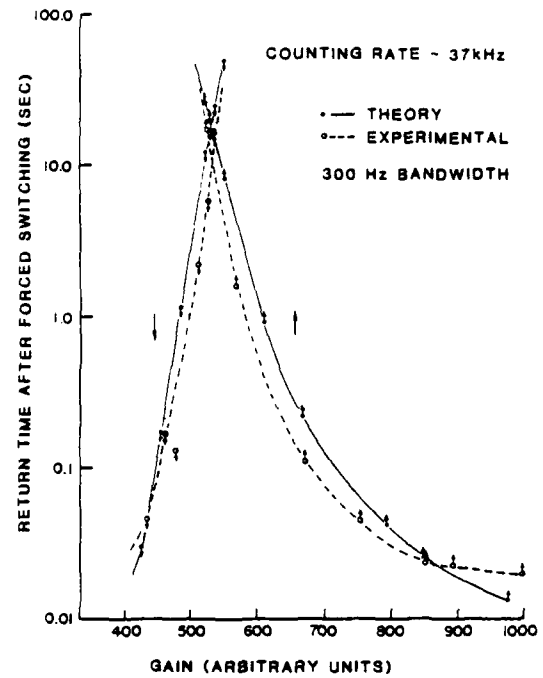
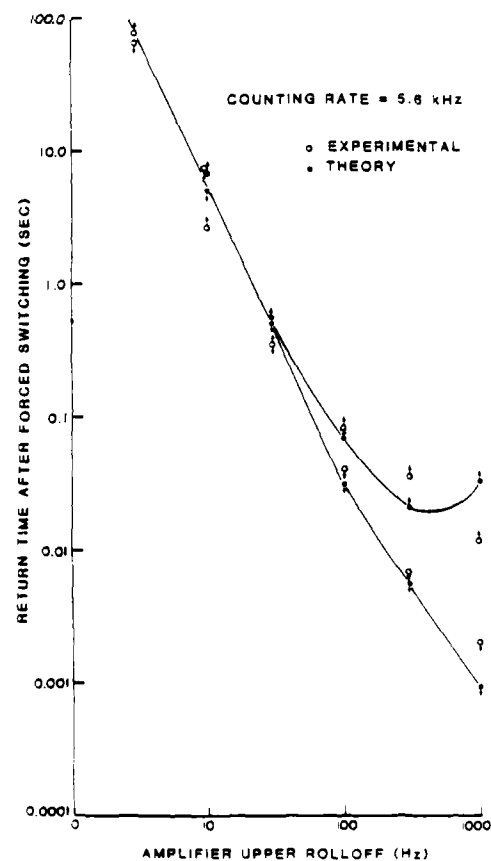


Figure 4. Return time after forced switching to the lower and upper states as a function of the amplifier upper rolloff frequency.



NOTES

NOTES

Room Temperature Optical Bistability in Semiconductors and Realization of Optical AND Gates

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Summary

Recent measurements of parameters concerning dynamic third order nonlinearity for both 77K and 300K near the band gap of InSb are reviewed. New measurements and theory of carrier lifetimes are now available.

Optical bistability at room temperature has been observed in a situation where two-photon excitation is the prime agent for populating states near the conduction band minimum.  $\chi^{(3)} \sim 10^{-4}$  e.s.u. is implied with a response time of 30 ns at an intensity of  $\sim 100 \text{ kW/cm}^2$ . Thus similar power levels to those seen in GaAs devices are demonstrated.

Using the more sensitive 77K  $5\mu\text{m}$  region where c.w. holding of the device near switch point is possible, AND gate operation is demonstrated and definite switching energies of  $\sim 5 \text{ nJ}$  determined. Delay of one of the switching pulses gives implicit information on carrier lifetime. Interaction of such logic elements over various path lengths will also be discussed.

## Advances in GaAs Bistable Optical Devices

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Bistable Optical Devices (BOD's) using GaAs as the nonlinear medium are viable candidates for the achievement of fast ( $\leq$  ns), room temperature,<sup>1</sup> low-power (mW), externally controllable<sup>2</sup> optical switches which are easily fabricated<sup>3</sup> and operated. Advances have been made in all of these areas and efforts are in progress to improve performances in ways that are simultaneously compatible.

Active external switching was performed at  $\approx 80$  K in an etalon consisting of  $4.1 \mu\text{m}$  of GaAs between  $0.21\text{-}\mu\text{m}$ -thick windows of  $\text{Al}_{0.42}\text{Ga}_{0.58}\text{As}$  dielectric coated to give  $\approx 90\%$  reflectivity. With a  $1\text{-}\mu\text{s}$ ,  $825\text{-nm}$ , triangular-shaped input pulse both switch-on and switch-off were about  $40$  ns. Faster switching was observed using a flat-topped input pulse with an intensity between the switch-on and switch-off values and a second  $600\text{-nm}$ ,  $10\text{-ps}$ ,  $1\text{-nJ}$  switching pulse. Switch-on occurred faster than the detector limit of  $200$  ps. The device was also externally switched off in  $\leq 20$  ns by a  $7\text{-ns}$ ,  $600\text{-nm}$ ,  $300\text{-nJ}$  pulse. This kind of switching off was a brute force technique in that the large pulse energy heated the etalon, shifting the hysteresis loop to higher intensities until the constant input intensity fell below the switch-off value. Both switch-on and switch-off were performed on a single input pulse as shown in Fig. 1. At room temperature a GaAs-AlGaAs multiple quantum well (MQW) structure was externally switched on with a  $\sim 10$  ps,  $800\text{-nm}$  pulse of only  $10$  pJ. Another MQW device which was previously bombarded with protons to speed up the response times showed for a  $200\text{-ns}$

triangular-wave input, switch-on in 5-ns and  $\sim 10$ -ns switch-off.

Room temperature bistability with modest input power was first achieved in a GaAs-AlGaAs MQW structure consisting of 61 periods of GaAs 336Å thick and 401Å AlGaAs layers. Removal of the device from the dewar allowed easier access and better focusing such that the contrast and difference between switch-up and switch-down intensities was better than had been observed previously (Fig. 2). More recently, devices containing MQW structures with 152Å thick GaAs layers have shown bistability with input powers of less than 20 mW.<sup>3</sup> The MQW devices exhibit regenerative pulsations<sup>4</sup> due to competing excitonic and thermal effects similar to the low-temperature GaAs. Several attempts have been made to heatsink the etalons; some show a significantly smaller thermal effect, but none yield cw bistability.

Fabrication of GaAs BOD's has almost become a routine operation. The MQW structures are grown on a GaAs substrate by molecular beam epitaxy<sup>5</sup>. This process in itself allows tremendous versatility in material properties as the layer thicknesses and/or doping schemes are varied. A combination of grinding and selective etching removes the substrate leaving the  $\sim 3$ - $\mu$ m thick MQW. The use of multiple etch-stop-layers has improved surface quality to a flatness of much better than  $\lambda/4$  over a few mm<sup>2</sup> area. Our unsupported samples are somewhat fragile, but handling techniques are being refined. The improved surface quality and the versatility in implementing these "flakes" into working devices is well worth the extra handling care required. It is quite easy to cement a flake to a washer allowing immediate study or evaporation of dielectric coatings. Flakes are also routinely mounted between dielectric mirrors saving considerable time and expense in Fabry-Perot BOD construction. These etalons have exhibited the low-power operation and external switch-on mentioned earlier. More complicated structures such as two overlapping flakes sandwiched between mirrors, and heatsunk devices have also been constructed. Further advances in etching techniques and refinements in etalon construction will no doubt greatly improve device performances.

Presently, when speed, size, power, temperature and fabrication are all considered, GaAs-AlGaAs MQW structures appear to be promising candidates for practical optical switching, but they are by no means optimized.

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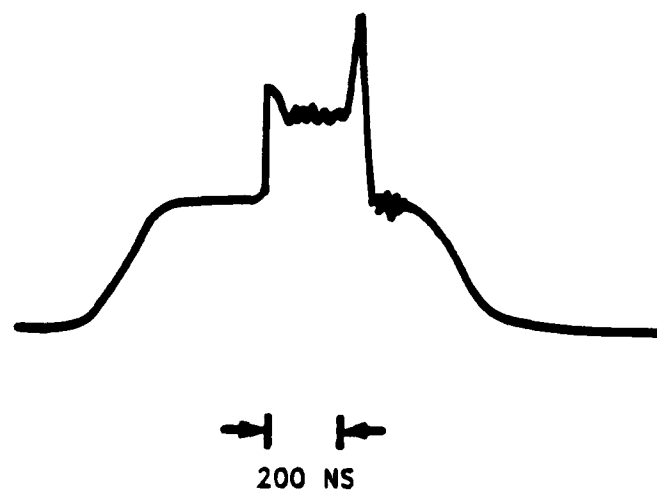


Figure 1. External switch-on and switch-off during a single input pulse.

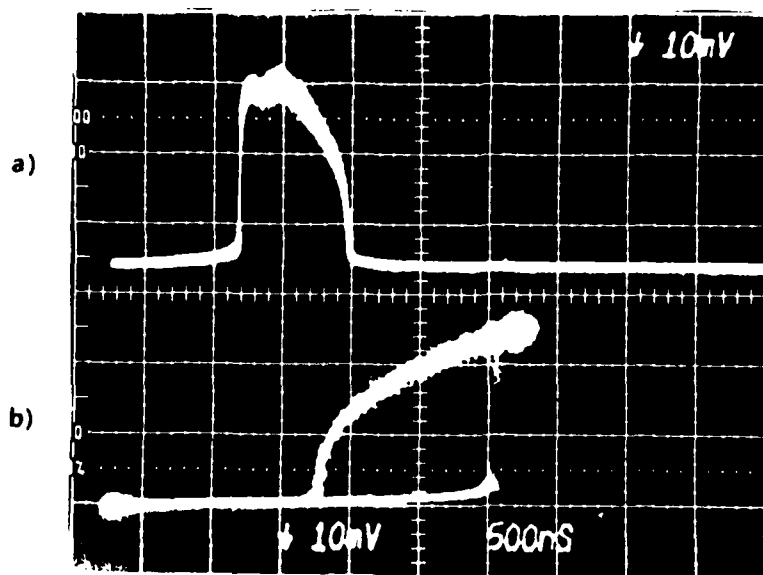


Figure 2. Room temperature optical bistability in a GaAs-AlGaAs MQW etalon.

a) Output vs. Time      b) Output vs. Input

## NONLINEARITIES AT THE BANDGAP IN InAs

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InSb has been shown to exhibit bistability and a very large nonlinear index.<sup>1</sup> A semiconductor which is similar, but which has a bandgap at shorter wavelengths is InAs. In this talk we present measurements of saturable absorption in InAs using an HF laser, which matches the bandgap of InAs at temperatures from 5° to 100° K. Using these measurements we have calculated a nonlinear refractive index of  $n_2 = 2 \times 10^{-5} \text{ cm}^2/\text{W}$  for light near but below the bandgap. These numbers are comparable to those measured in InSb and indicate that InAs may be a good material for bistable devices.

Figure 1 shows a plot of bandgap vs. temperature for InAs with relevant HF laser lines indicated. By changing the temperature of the sample, the bandgap can be tuned to a particular HF line, allowing measurements at seven different wavelengths over a wide temperature range.

Transmission measurements were made on polished, single crystal InAs samples ( $N_D - N_A = 3 \times 10^{16} \text{ cm}^{-3}$ ). Data were taken on crystals of 130 and 70  $\mu\text{m}$  thickness. The HF laser was cw, grating-tunable with an average power of 30mW and a single spatial mode. The output from this laser was focused down on the sample using a  $\text{CaF}_2$  lens, giving a rough focal spot size of 40  $\mu\text{m}$ . Input and output powers were measured using InAs photodiodes with the output beam focused down onto

the output detector in order to measure total power. The linearity of the photodiodes was checked by removing the sample. Figure 2 shows the total power transmitted vs. the total power incident on a 70  $\mu\text{m}$  thick sample at three different temperatures and three different wavelengths. A semi-empirical fit to the data points was done by assuming a gaussian beam and an absorption coefficient given by  $\alpha = \alpha_0 (1 + I/I_s)^{-1}$ . The saturation intensity obtained for each set of data is shown.

The measured values for saturation intensity are comparable to those measured in InSb.<sup>2</sup> As the temperature was changed, the wavelength of observation was also changed, so that the distance into the bandgap was roughly comparable. It can be seen that the saturation intensity was also comparable. Estimates of maximum temperature rise at the beam spot were less than 1°K for the powers used in this experiment. Furthermore, if heating occurred, the absorption should increase with intensity, ruling out the possibility of thermal effects causing the saturable absorption.

From the measurement of saturable absorption within the bandgap, one can estimate the nonlinear index which will be measured below the bandgap. Following a procedure similar to that used by Moss,<sup>3</sup> we may obtain an equation for the nonlinear index in terms of the measured saturation intensity:

$$n_2 = \frac{\hbar c \alpha}{2\pi I_s E_g} \ln \left( \frac{2E_g}{E_g - \hbar\omega} \right)$$

where  $E_g$  is the energy of the bandgap and  $\hbar\omega$  is the energy of the applied light. The linear absorption coefficient is  $\alpha$ .

At  $T = 30^\circ\text{K}$  and for a light frequency given by  $(E_g - \hbar\omega)/E_g = 1/400$ , linear absorption measurements give  $\alpha = 100\text{ cm}^{-1}$ . Thus, since  $I_s = 250\text{ W/cm}^2$ ,  $n_2 = 2 \times 10^{-5}\text{ cm}^2/\text{W}$ . This is comparable to the values measured in InSb.

Experiments are underway to look for optical bistability by temperature tuning the bandgap to optimize the nonlinear index.

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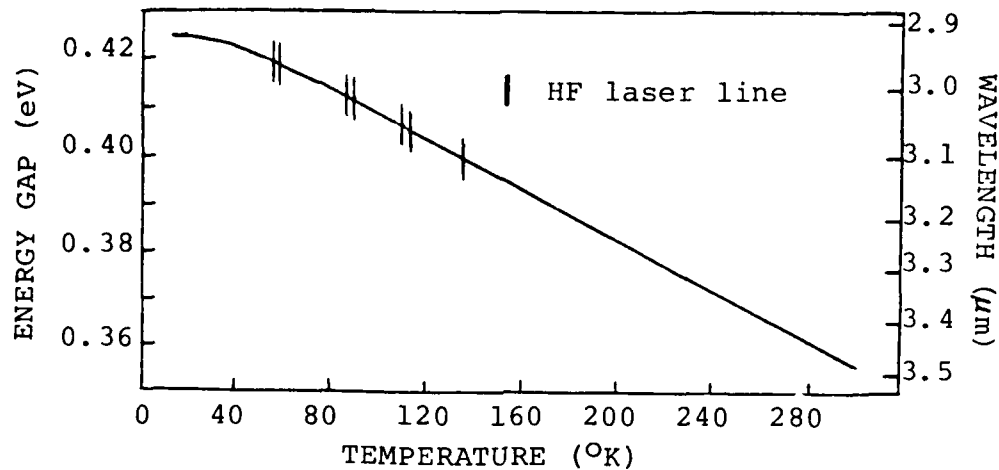


Figure 1. InAs bandgap vs temperature with the HF lines indicated.

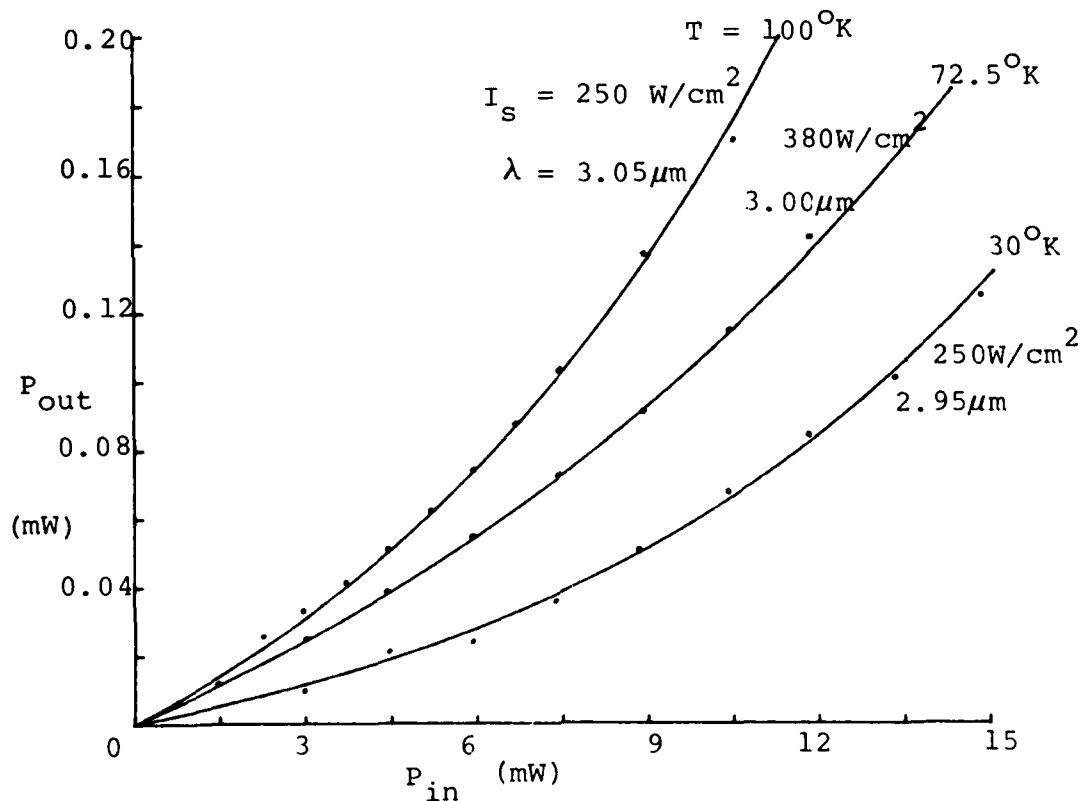


Figure 2. Power out vs. power in for three different wavelengths at three different temperatures in InAs. The points are the experimental measurements and the solid curves represent a semi-empirical fit to the data giving the stated saturation intensities.

Nonlinear Optical Interfaces

by

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SUMMARY

In this paper we will review the results of previous theoretical and experimental studies of nonlinear interfaces, and report new experimental results obtained using a liquid suspension of dielectric particles<sup>(1)</sup> as the nonlinear medium. This medium has a sufficiently large nonlinearity to permit nonlinear interface experiments to be performed with low-power CW laser beams, and thus simple, direct experimental verification of theoretical predictions is now possible for the first time.

Early theoretical work on nonlinear interfaces (interfaces between two dielectric materials, one of which has an intensity - dependent refractive index) predicted bistable behavior for incident plan waves.<sup>(2)</sup> The first experimental results appeared to exhibit optical bistability although the detailed behavior did not conform to the plane-wave predictions.<sup>(3)</sup>

We recently developed a numerical technique to study the behavior of a (two-dimensional) Gaussian light beam at a nonlinear interface. The results of this analysis show several new features. The most dramatic deviations from the plane-wave results, however, are the prediction of a second reflectivity 'step' as the input power is increased, and the absence of hysteresis upon subsequent reduction of the input power. These results are shown in Figure 1.

We performed experiments using a CW argon ion laser beam with a LiF crystal ( $n=1.39$ ) as the linear medium and an aqueous suspension of 600 Å quartz particles ( $n=1.37$ ) as the nonlinear medium. The experimental conditions were

chosen to correspond to the parameters used for the numerical analysis. The experimental results are also shown in Figure 1. These results conform much better to the 'Gaussian beam' curve than the 'plane-wave' curve. Note, however,

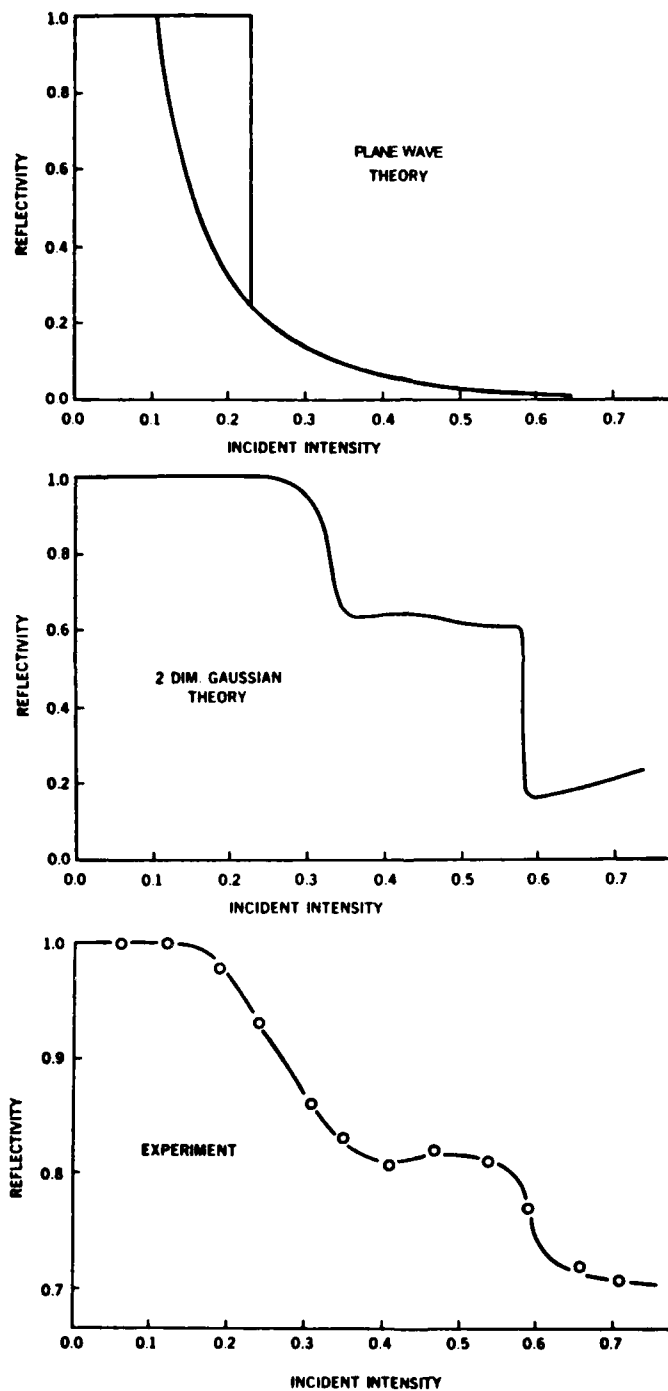


Figure 1 Reflectivity vs. incident intensity for incident angle equal to  $0.7 \times$  critical angle.

that the reflectivity 'steps' are smaller than the (two-dimensional) Gaussian beam predictions. The critical intensity for the first and second reflectivity 'steps' is in good quantitative agreement with the results of our analysis.

For some experimental conditions, hysteresis was observed. Only the upper branch of the reflection hysteresis characteristic was stable, however. The lower branch would decay to the upper branch with a time constant of  $\sim 10^3$   $\times$  the response time of the nonlinear medium.

It is interesting to take a new look at the experimental data for the glass - CS<sub>2</sub> interface<sup>(3)</sup> and to reinterpret it in terms of our present understanding of nonlinear interface behavior. The experimental data shows that at intensities substantially above the first reflectivity 'step' there is a second 'step'. These results appear to be in qualitative agreement with our numerical analysis. The hysteresis observed in these early experiments is also not inconsistent with our prediction of an unstable lower state, for the measurement time was of the order of 300 $\times$  the response time of the nonlinearity (2ps) and our current experiments show that this is of the order of the stability time of the lower state.

What are the implications of our results for device applications? Nonlinear interface devices are attractive because of their simplicity, and their potential for sub-picosecond response (with suitable fast nonlinear materials). The quasi-stability of the lower branch could cause problems. Gaussian beam devices do not have a high contrast between "on" and "off" states. Because a high contrast is predicted from the two-dimensional Gaussian beam analysis, we would expect that the contrast could be much improved by using an elliptical input beam.



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Mirrorless Intrinsic Optical Bistability Due to the Local  
Field Correction in the Maxwell-Bloch Formulation

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Summary

The local field correction (LFC) is associated with the passage from microscopic to macroscopic electrodynamics. The conventional derivation of the LFC is found in Jackson<sup>1</sup>, and the implication is that the correction is due to all of the atoms in the system, which implies some kind of collective phenomenon. In the work reported here, we draw heavily upon the discussion of the LFC of Van Kranendank and Sipe<sup>2</sup> (KS).

In the passage from microscopic to macroscopic electromagnetism, two main problems occur: (1) The macroscopic Maxwell's equation in isotropic media is

$$-\nabla^2 E + \frac{\partial^2 E}{\partial t^2} = \frac{4\pi}{c^2} \frac{\partial^2 P}{\partial t^2} \quad (1)$$

where  $P$  is the macroscopic polarization. The solution of the macroscopic field  $E$  from (1) then results in contributions to the total electric field

all the atoms. If this field is used to drive, say atom  $\ell$ , it contains the self-field of atom  $\ell$ . As shown by  $KS^2$ , this extra contribution is removed by taking the atom to be driven by the local field  $E_L$ ,

$$E_L = E + \frac{4\pi}{3} P, \quad (2)$$

where the amplitude of  $P$  is given by the amplitude of atom  $\ell$ , and not by the surrounding atoms. Thus, in this interpretation, the local field seems to be a single atom phenomenon. (2) In the case of crystals, it is by no means clear that the field at atom  $\ell$  is accurately represented by smearing out positions of atoms  $j \neq \ell$ . This can only be significant for atoms in the neighborhood of  $\ell$ , so the correction is again proportional to  $P$  in the static limit (there are other problems in the dynamic limit). If one can neglect these other problems, then the LFC is given as

$$E_L = E + \left( \frac{4\pi}{3} + s \right) P \quad (3)$$

where  $s$  is a structure factor. In this situation, the contribution from  $s$  appears to be collective (it may not be cooperative).

For the purposes of this study, we assume that (2) is the appropriate relation and incorporate this LFC in the Maxwell-Bloch model representing a collection of two-level atoms interacting with an externally-applied coherent field. The results of numerical calculations are presented for the case of a small volume of nearly resonant absorbing medium (i.e., a volume smaller than a resonant wavelength) in one-spatial dimension. The usual slowly-varying envelope and phase approximations are not valid for this case and the conditions on the field and its derivatives at the dielectric boundaries are explicitly satisfied. It is found that for suitable values for the material parameters, a bistable relation exists between the applied incident field and the output field, which is due entirely to the local field correction.

Bowden, Hopf, and Louisell

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NOTES

# NOTES

Optically Induced Changes in the  
Faraday Rotation Spectra of the Ferromagnetic  
Semiconductor  $\text{CdCr}_2\text{Se}_4$ \*

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Chromium chalcogenide spinel semiconductors have gained attention because of the strong interaction between their electronic and magnetic systems.  $\text{CdCr}_2\text{Se}_4$ , the most extensively studied Cr spinel, is ferromagnetic and orders at 130 K. The optical absorption edge is found to red shift for temperatures below the Curie point<sup>(1)</sup>. Faraday rotation spectra measured near the absorption edge are highly dispersive with peak rotation increasing and shifting to the red as temperature is reduced. In particular, the rotation spectra scanned at fixed temperature display zero crossings characteristic of the temperature ( $T$  less than  $T_C$ ). The red shifting of both the absorption edge as well as the rotation dispersion result from increased magnetic order with reduced temperature<sup>(2)</sup>.

The Faraday rotation spectra were measured with a tunable probe beam. A dramatic shift in the spectra could be observed when measurements were made with simultaneous application of a weak monochromatic pumping beam. Each were directed in a coaxial fashion through a sample suspended in the bore of a split coil superconducting magnet configured for optical access. Right circularly polarized pumping at 1.06 microns effectively shifts the rotation dispersion to the red as if the sample were cooled. Left circularly polarized light had the opposite effect. The stability of the probe beam polarization, as it traverses the sample, is thus seen to depend upon the sense of pump polarization. The maximum response is found at probe wavelengths near rotation zero crossings. In these regions, the slope of the rotation dispersion is extremely steep and a small offset induced by the pump produces a large change in probe beam

Faraday rotation. At 78 K and pumping with a power of .12 milliwatts, the rotation spectrum can be shifted  $\pm 30^\circ$  depending upon the sense of pump polarization. Such a shift brings about a variation in probe Faraday rotation by  $\pm 5$  degrees.

The effects described above are best explained within the context of optically induced magnetization. For  $\text{CdCr}_2\text{Se}_4$  the possibility of light enhancing magnetic order has been raised before<sup>(3)</sup>. Previous experiments have detected transient induced magnetization due to high powered pump excitation<sup>(4)</sup>. The sensitivity afforded by the all-optical technique presented here permits very low pump power to be effective, in addition, heating effects and transient artifacts are discriminated against. Selective repopulation of magnetically active bands by the pumping radiation is found to account for the observations. Details of the opto-magnetic model can be found in reference 2.

\* Author acknowledges support of GTE Labs during his tenure as visiting graduate research fellow. Work also supported in part by AFOSR contract F49620-C0182.

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## Multi-parameter universal route to chaos in a Fabry-Perot resonator

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We recently reported [1] preliminary investigations of positive-branch instabilities in nonlinear resonators in the Fabry-Perot (folded) configuration. We deal with the dispersive limit of the Maxwell-Bloch equations (Kerr medium). The finite longitudinal relaxation time ( $\tau$ ) means that diffusion of the excitation is possible, and we assume that the diffusion length greatly exceeds the wavelength. We thus derive the following equations for the nonlinearity, the forward and the backward fields:

$$\tau \dot{\chi}^{NL} = -\chi^{NL} + |F|^2 + |B|^2 \quad (1)$$

$$\frac{\partial F}{\partial x} + \frac{n_0}{c} \dot{F} = i \chi^{NL} F - \frac{\alpha}{2} F \quad (2)$$

$$-\frac{\partial B}{\partial x} + \frac{n_0}{c} \dot{B} = i \chi^{NL} B - \frac{\alpha}{2} B \quad (3)$$

These partial differential equations are integrated directly to obtain the numerical results quoted below, but an integration along characteristics leads to a quasi-difference equation form which is a generalisation of the result obtained by Firth [2]

$$F(0, t + t_R) = A + F(0, t) (R_1 R_2)^{1/2} e^{-\alpha L} e^{i\theta^{NL}} \quad (4)$$

where

$$\theta^{NL} = \int_{-\infty}^t \frac{dt'}{\tau} e^{-(t-t')/\tau} \{ |F(0, t')|^2 (1 - e^{-\alpha L} + R_2 e^{-\alpha L} - R_2 e^{-2\alpha L}) / \alpha L \\ + \int_0^1 dy e^{-\alpha L(1-y)} [ |F(0, t' + yt_R)|^2 + R_2 e^{-\alpha L} |F(0, t' - t_R + yt_R)|^2 ] \} - \phi_0 \quad (5)$$

The stationary solutions of (4,5) for particular parameter values are shown in Fig. 1, showing e.g., eight positive slope branches at  $A = 4.463$ . The stability of the stationary solutions is well described by a linearisation of (4): Fig. 2 shows how the positive-branch instability regions narrow down and disappear as the ratio  $\tau/t_R$  is increased. Even when this ratio is as large as 5, however, Fig. 3 shows that only one of the eight branches is stable: initial conditions corresponding to all other branches lead, after numerical integration over a few hundred times  $t_R$ , to limit cycles in different regions of the phase plane. We show that all orbits, fixed points and strange attractors must be within an annulus of the phase plane of outer radius  $AB/(1-B)$  where  $B$  is the feedback fraction [3], equal to 0.3 in this case. The inner radius is smaller by a factor  $(1-2B)$ , vanishing if  $B \geq \frac{1}{2}$ .

We have examined in detail the development of an instability on the lowest branch (stable point in Fig. 3). As  $\tau$  is reduced, an oscillation with period  $\approx 2.5 t_R$  develops for  $\tau \sim 2t_R$ : further reduction in  $\tau$  results in a period-doubling sequence leading to chaos at  $\tau \sim 1.8 t_R$ .

As shown in Table , the bifurcation sequence closely obeys the universality relation found by Feigenbaum [4], i.e. the response time  $\tau_n$  at the  $n^{\text{th}}$  bifurcation obeys the relationship

$$\frac{\tau_{n-1} - \tau_n}{\tau_n - \tau_{n+1}} = \delta_n \rightarrow \delta_\infty = 4.669 \quad (6)$$

Investigation reveals that the same universal sequence occurs when the limit point is approached by varying the input field  $A$  or the detuning  $\phi_0$  - Table - or, presumably, any other parameter. Further, the form of (6) means that any linear combination of parameters obeys the same relation, so that a limit point may be approached along any convenient "line" in parameter space.

A major difficulty with numerical investigation of bifurcation sequences in

equations as complex as (1-3) is that the convergence of time series becomes poor as a bifurcation point is approached: simple analysis points to an exponential convergence in general, but only a geometric convergence at the bifurcation point. We have thus used an extrapolation procedure to locate the bifurcations. If the vector of "new" Fourier components simply re-scales as the control parameter  $\lambda$  is increased beyond the bifurcation point  $\lambda_n$ , simple analysis shows that the scale factor must be  $(\lambda - \lambda_n)^{1/2}$ . Fig. 4 shows that this rule is excellently obeyed, the squared differences ( $\Delta^2$ ) between corresponding intensity maxima showing an excellent linear dependence, with all lines extrapolating to zero at essentially the same point. This technique is used to obtain the data in the Table. (The same technique has recently been applied to the high-finesse ring resonator [5]: that work is a special case of our own [1]).

Control Parameter $\lambda$	$\lambda_8: P4 \rightarrow P8$	$\lambda_\infty$	$\delta_8$	$\delta_{16}$	$\delta_{32}$
A	4.476	4.463	3.902	4.768	4.612
$\phi_0$	.09046	0	4.411	4.601	4.658
$\tau/t_R$	1.826	1.78636	4.506	4.643	4.618

We believe that (1-3) are among the most complex systems in which Feigenbaum universality has been found, and we will present an analysis of a general class of equations, including (4,5) which already yields in lowest order a value 4 for the parameter  $\delta$ .

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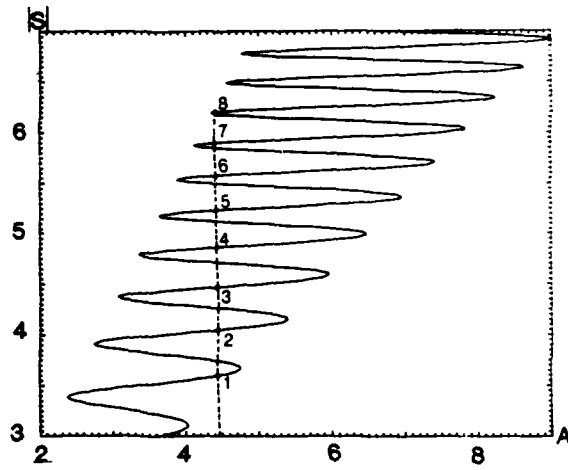


Fig. 1

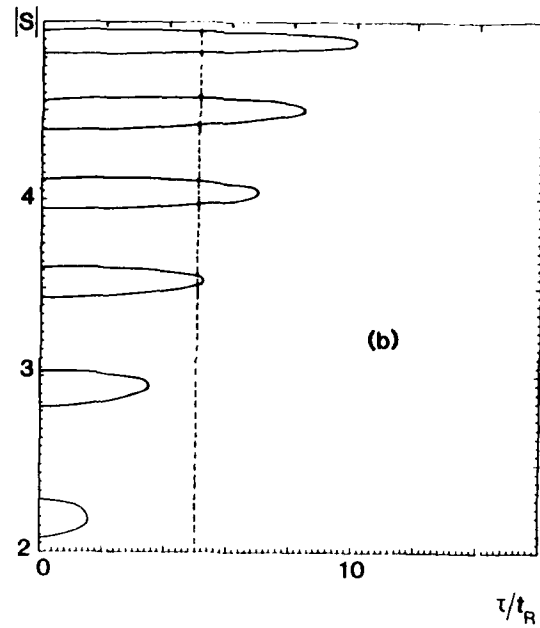


Fig. 2

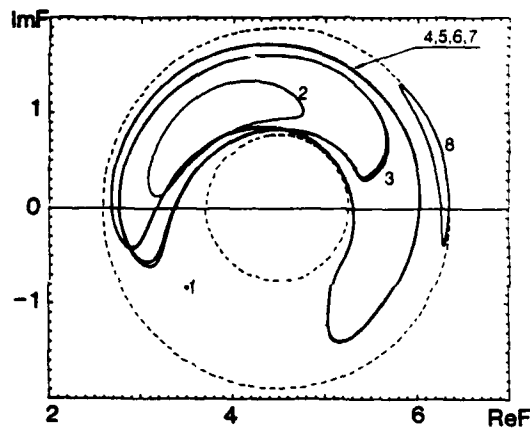
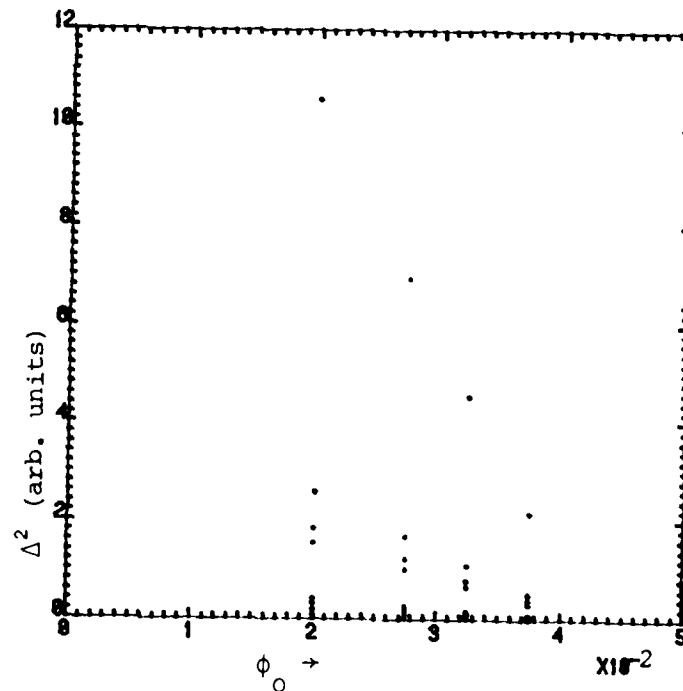


Fig. 3

Fig. 4



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#### SUMMARY

This paper describes a theoretical analysis of the steady-state and transient behavior of a laser-with-saturable-absorber system consisting of two GaAlAs semiconductor laser elements in an external cavity.

The facets of each semiconductor laser are coated so that when placed in the cavity and properly biased, one functions as a gain medium and the other as a saturable absorber. The guided-wave structure of the absorber section is used to good advantage to provide the high power densities necessary for saturation. Since the two structures are electrically isolated, the problem of a parasitic resistance encountered in double-section diode-lasers<sup>1</sup> is eliminated.

The analysis is based on three coupled, semiconductor-laser, rate-equations: one for the photon density in the cavity, a second for the electron concentration in the gain section, and a third for the electron concentration in the absorber section. The steady-state analysis leads to a simple result for  $N_1$ , the upper state photon density at the low-to-high output switching point:

$$N_1 = N_s [(R/R_{crit}) - 1], \quad (1)$$

where  $N_s$  is the saturation photon-density;  $R = g_A N_0$ , with  $g_A$  the constant of proportionality in the assumed linear relationship between absorption and electron density, and  $N_0$  the carrier density required for transparency in the absorber element; and  $R_{crit}$  depends in a simple way on the photon cavity lifetime  $\tau_{ph}$ , the spontaneous carrier lifetime  $\tau_{sp}$ , the cavity length  $L$ , and  $N_s$  (defined above). It also leads to a simple result for  $N_2$ , the upper state photon-density at the high-to-low output switching point:

$$N_2 = N_s [R/R_{crit}]^{\frac{1}{2}} - 1]. \quad (2)$$

Similar formulas have been derived by Lugiato et al<sup>2</sup> using a semiclassical treatment.

To study the transient response of the system, the rate-equations were solved numerically for various types of switching pulses. We find that bistable operation is achieved when either (or both of) the absorber section or the gain section is activated by a suitable electrical pulse. In addition, our results predict ringing with fast-risetime switching pulses and

critical slowing down near the current for switching the cavity to its high output state.

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## Optically-Induced Bistability in a Nematic Liquid Crystal

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Since 1979, considerable effort has been expended by many researchers in analyzing the purely optical-field-induced molecular reorientation in a homeotropically oriented nematic liquid crystal (NLC) by a normally incident light wave.<sup>1-4</sup> Recently, the exact solution which is consistent with Maxwell's equations for describing the orientation of the molecule was obtained by Ong.<sup>5</sup> In this paper, we shall discuss the exact solution describing the orientation of the director. In particular, by examining the deformation angle near the threshold, the criterion for the existence of a first-order Freedericksz transition is obtained.

We consider a homeotropically oriented NLC cell of thickness  $d$  confined between the planes  $z=0$  and  $z=d$  of a Cartesian coordinate system. The NLC director always lies in the  $xz$ -plane and in the absence of a light beam, the directors are parallel to the  $z$ -axis everywhere.  $\theta(z)$  denotes the angle between the director and the  $z$ -axis. Then the director can be described by  $\hat{n}(\vec{r}) = (\sin \theta, 0, \cos \theta)$ . A harmonic time-dependent light beam is normally incident on the NLC medium with the polarization parallel to the plane of incidence, which is the  $xz$ -plane.

By taking the time average of the energy flow  $\partial F_{\text{Opt}}/\partial t + \text{div } \vec{S} = 0$ , we find that  $\text{div } \langle \vec{S} \rangle = 0$ , where  $F_{\text{Opt}}$  is the electromagnetic energy density and  $\vec{S}$  is the Poynting vector of the optical field. Consequently the time-average of the  $z$ -component of the Poynting vector is a constant throughout the medium. This conclusion agrees with the geometrical optics approximation<sup>3</sup> but disagrees with the infinite-plane-wave approximation<sup>4</sup> in which the magnitude of the Poynting vector is a constant. If the scattering loss in traversing



the medium can be neglected, then  $\langle S_z \rangle$  is equal to the intensity of the incident beam,  $I$ .

It can be shown that for a normally incident wave,  $F_{\text{opt}} = S_z n_p / c$  with  $n_p = n_o n_e / (n_o^2 \sin^2 \theta + n_e^2 \cos^2 \theta)^{1/2}$ , where  $n_o$  and  $n_e$  are the ordinary and extraordinary refractive indices respectively at the optical wavelength  $\lambda$ .<sup>5</sup> The free energy density of the NLC can then be written as

$$F = \frac{1}{2} k_{11} (\text{div } \hat{n})^2 + \frac{1}{2} k_{22} (\hat{n} \times \text{curl } \hat{n})^2 + \frac{1}{2} k_{33} (\hat{n} \cdot \text{curl } \hat{n})^2 - \frac{I}{c} n_p(\theta) \quad (1)$$

where  $k_{11}$ ,  $k_{22}$ , and  $k_{33}$  are the splay, twist and bend elastic constants. By the symmetry of the problem, we look for solutions which are symmetrical w.r.t. the  $z = d/2$  plane, i.e.  $\theta(z) = \theta(d-z)$ . Then minimization of the total free energy  $\int F d^3 \vec{r}$  leads to the Euler equation which has a solution of the form

$$z = \left( \frac{ck_{33}}{2I} \right)^{1/2} \int_0^\theta \left[ \frac{1 - k \sin^2 \theta}{n_p(\theta_m) - n_p(\theta)} \right]^{1/2} d\theta, \quad (2)$$

for  $\frac{d\theta}{dz} \neq 0$  and  $0 < z < d/2$ , where  $k = (k_{33} - k_{11}) / k_{33}$  and  $\theta_m = \theta(z=d/2)$ . In obtaining Eq. (2), we have made use of the rigid boundary conditions at the two interfaces, i.e.  $\theta(z=0) = \theta(z=d) = 0$ .

We compute the solution for the tilt angle up to and including terms  $\sim \theta^2$  and obtain the following equation for the tilt angle:

$$\theta \sim \theta_m \sin(\pi z/d) \quad (3)$$

where  $\theta_m^2 = (\sqrt{I/I_{\text{th}}} - 1) / B$ ,  $I_{\text{th}} = c k_{33} (\epsilon_{\parallel}/n_o \epsilon_a) (\pi/d)^2$ , where  $I_{\text{th}}$  is the threshold intensity,  $B = (1-k-9u/4)/4$ ,  $u = \epsilon_a/\epsilon_{\parallel}$ ,  $\epsilon_{\perp} = n_o^2$ ,  $\epsilon_{\parallel} = n_e^2$ , and  $\epsilon_a = \epsilon_{\parallel} - \epsilon_{\perp}$ . As a comparison, infinite-plane-wave approximation predicts that  $I_{\text{th}} = ck_{33} (\epsilon_{\parallel}^2/n_o \epsilon_{\perp} \epsilon_a) (\pi/d)^2$  and  $B = (1-k-9w/4-3u/4)/4$  where  $w = 1 - (\epsilon_{\perp}/\epsilon_{\parallel})^2$ .<sup>4,5</sup>

From Eq. (3), we see that as  $B > 0$ , the transition is second-order. However, for  $B < 0$ , small distortion are not stable and the transition becomes a first-order transition accompanied by hysteresis. Thus, the criterion for the existence of the first-order optically-induced Freedericksz transition is

given by  $B < 0$ , i.e.  $k_{11}/k_{33} + 9 \epsilon_{\perp}/4 \epsilon_{\parallel} < 9/4$  for this approach, and  $k_{11}/k_{33} + 3 \epsilon_{\perp}/4 \epsilon_{\parallel} + (3 \epsilon_{\perp}/2 \epsilon_{\parallel})^2 < 3$  for the infinite-plane-wave approximation. The criterion for the existence of a first-order transition is shown in Fig. 1. For the first-order transition, the lower threshold intensity  $I'_{th}$  can be determined by  $dI/d\theta_m=0$ . By computing the integral in Eq. (2) up to and including terms  $\sim \theta^4$ , we find that  $I'_{th}=I_{th}(1-B^2/4G)^2$  and  $\theta_m^2 = [-B+(B+4GF)^{1/2}]/2G$ , where  $F = (\sqrt{I/I_{th}} - 1)$  and  $G = 11/2-k+9u/4+63ku/4-9k^2/2-26lu^2/32$ . Infinite-plane-wave approximation predicts that  $G = 11/2-k+9w/4+3u/4+63kw/4+3ku+153wu/16-9k^2/2-26lw^2/32-189u^2/32$ .<sup>5</sup>

By examining the material parameters from known NLC's, we notice that the bistability could be observed experimentally for PAA (p-azoxyanisole) in a temperature range of 110 to 130°C and a pump beam of intensity  $\sim 150$  Watts/cm<sup>2</sup> for a cell of 250  $\mu$ m thick. Figure 2 shows the predicted maximum deformation angle as a function of the intensity for PAA and MBBA [N-(p-methoxybenzylidene-p-butyraniline)]. For MBBA, the transition is second-order but for PAA the transition is first-order accompanied by hysteresis. This hysteresis, if it is confirmed experimentally, may be exploited in bistable display systems.

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Figure 1

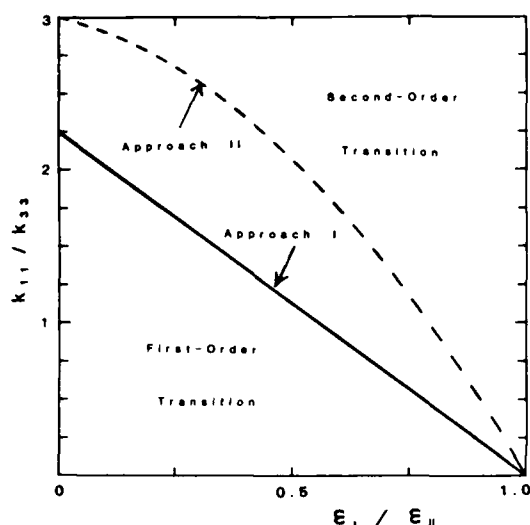


Figure 2

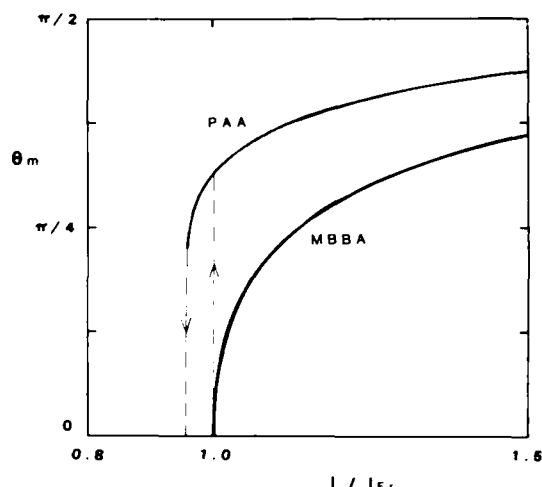


Figure 1. The criterion for the existence of a first-order optically-induced Freedericksz transition, where this approach is referred to as approach I and the infinite-plane-wave approximation is referred to as approach II.

Figure 2. The maximum deformation angle as a function of the pump beam intensity for MBBA and PAA. For MBBA, we put  $\lambda = 6328 \text{ \AA}$ ,  $n_o = 1.544$ ,  $n_e = 1.758$ ,  $k_{11}$  and  $k_{33} = 6.95, 8.99 \times 10^{-7} \text{ dyn}$  respectively. The Freedericksz transition is second-order with  $B=0.06$ ,  $G=0.06$ , and  $I_{Fr}=120.6 \text{ W/cm}^2$  for a cell of  $250 \text{ \mu m}$  thick. For PAA, we put  $\lambda=4800 \text{ \AA}$ ,  $n_o=1.595$ ,  $n_e=1.995$ ,  $k_{11}$  and  $k_{33} = 9.26, 18.10 \times 10^{-7} \text{ dyn}$  respectively. The Freedericksz transition is first-order accompanied by hysteresis with  $B = -0.08$ ,  $G = 0.07$ ,  $I_{Fr}$  and  $I'_{Fr} = 149.0$  and  $142.8 \text{ W/cm}^2$  respectively for a cell of  $250 \text{ \mu m}$  thick.

## Room-Temperature Thermal Optical Bistability in Thin-Film Interference Filters and Dye-Filled Etalons

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Passive optical bistability in semiconductors was first seen in an interference filter with an intermediate layer of ZnS one-wavelength thick, and implied switch-on time of 200  $\mu$ s.<sup>1</sup> Thin-film interference filters may be potential practical bistable devices due to their room-temperature operation, extreme thinness, ease of production, and low-power operation.

We have observed thermal optical bistability at mW powers in narrow-band visible interference filters with ZnS and ZnSe spacers. Thermal optical bistability can be defined as dispersive optical bistability<sup>2</sup> in which the intensity dependence of the optical path length arises from a temperature change of the intracavity medium. The slow switching times (200  $\mu$ s at best), the sign of the nonlinearity ( $n_2 > 0$ ), and the laser-etalon detuning are consistent with a thermal mechanism.

Proper input focusing was critical. In general the bistability loop drifted in time, preventing CW operation. Apparently as the filter heated up, the cavity progressively detuned, causing the switch-on intensity to increase in time. Also, in a ZnS 514.5-nm filter (1.2-nm FWHM) an irreversible change in transmission and shift of the peak by 1.2-nm to shorter wavelength was observed. This transmission-peak shift took several months to recover, suggesting that the local water vapor concentration was changed via laser "annealing."

Further work is on-going to investigate production conditions of thin films to determine whether fast switching (implied to be  $\approx$  ns, due to a thin-film nonlinearity apparently caused by structural effects<sup>1</sup>) is achievable.

Perhaps the simplest device exhibiting thermal CW optical bistability is dye solution between two mirrors. The addition of Rhodamine B to ethylene glycol or other solvents lowers the bistability turn-on intensity compared with that for solvent alone. The dye aids by increasing the absorption, heating the solvent, and lowering the solvent index of refraction. Various solvents and dyes at different concentrations were tried; all had similar switch-on times from 200 to 20  $\mu$ s, strongly dependent on the input focusing. Input powers were 5 mW to 2 W.

If the input laser wavelength lies within a strong dye absorption band (or within a weak band, but the focus is at the edge of a bubble), the bistability regeneratively pulsates.<sup>3</sup> The oscillation period varied from a few milliseconds to seconds.

We have investigated crosstalk and observed that two bistable regions on the same etalon can operate independently within three to four spot diameters apart, apparently limited by thermal diffusion. Deliberate control of a red laser beam by a more powerful yellow laser was possible when the two beams overlapped, as the etalon could not simultaneously transmit both wavelengths. The simplicity of this device and flexibility in choice of lasers should encourage widespread use in demonstrations, student labs, and studies of optical signal processing.

CW dye etalon bistability and the resultant far-field output will be demonstrated at this meeting.

We gratefully acknowledge support from NSF, AFOSR, and ARO.

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Self-Defocusing and Optical Crosstalk  
in a Bistable Optical Etalon

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Self-focusing effects in optical bistable hysteresis loops have been observed in a GaAs-GaAlAs superlattice.<sup>1</sup> Experimentally, one sees drastic overshooting in the hysteresis loops and changes in the output profile as the imaging lens is translated along the propagation direction, thus imaging different object planes onto a fixed image plane (i.e. aperture). Nearly identical results are found by a one-transverse-dimension numerical simulation of good-cavity bistability in which the output is free-space propagated backwards in space, corresponding to the movement of the imaging lens. These effects arise from a radially-dependent phase shift caused by the excitonic nonlinear refraction and a gaussian beam profile.

The parallel operation<sup>2</sup> of neighboring optical bistable devices on the same etalon is also simulated numerically in both one- and two-transverse-dimension cases. Diffractive coupling can cause a device operating in the lower branch to switch to the upper branch when its neighbor is switched on. Simulations show, for Fresnel numbers close to unity, that a separation of a few beam widths is sufficient for independent operation. The results give promise for performing parallel operation, in spite of transverse diffractive coupling. Transverse effects which are neglected in the calculation, such as exciton and free-carrier diffusion, could make larger separations necessary for independent operation.

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# Routes to Optical Turbulence in a Dispersive Ring Bistable Cavity Containing Saturable Nonlinearities

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The transition to turbulence in fluid dynamical systems involves many different pathways depending on parameters such as the Reynolds number, aspect ratio of the fluid, etc. Remarkable progress has been made over the past 10 years in understanding qualitatively, transitions to low level turbulence in low aspect ratio, low Reynolds number systems by studying mappings of one and two dimensional nonlinear functions.<sup>1</sup> However no quantitative relation appears to exist between fluid parameters (Reynolds #, Prandtl # etc.) and the parameters appearing in the mappings.

Numerous papers citing period doubling bifurcations in nonlinear optical systems have appeared since Ikeda's original paper in 1979. With only two exceptions,<sup>2,3</sup> all theoretical studies have been confined to plane wave analyses. These studies have focussed on establishing that Feigenbaum sequences exist for instabilities in these systems. Our contention is that the Feigenbaum period doubling sequences represent a subset of a much more general class of instabilities.

As an explicit example, we study a unidirectional ring bistable cavity containing a nonlinear saturable medium. In the plane wave approximation and in the limit that the nonlinear medium response is much faster than a cavity roundtrip time  $t_R (\equiv L/c, L = \text{total cavity length})$ , the dynamics of the complex intra-cavity field reduces to the following map

$$E_{n+1} = rT E_{in} + B \exp \left[ \frac{\alpha |E_n|^2}{1 + \Delta^2 + |E_n|^2} \right] \exp[i\delta(|E_n|^2)] E_n \quad (1)$$

where

$$\delta(|E_n|^2) = \psi - \alpha L \Delta \left[ 1 - \frac{|E_n|^2}{1 + \Delta^2 + |E_n|^2} \right]$$

is the intensity dependent total cavity mistuning. Eq. (1) maps the field ( $E_n$ ) at roundtrip  $nt_R$  to the field ( $E_{n+1}$ ) a cavity roundtrip later ( $(n+1)t_R$ );  $E_{in}$  is the input amplitude (all amplitudes are scaled to the saturation amplitude),  $B = Re^{-\alpha L}$  ( $\alpha L = \alpha_0 L/2(1 + \Delta^2)$ ;  $\Delta = (\omega - \omega_{ab})/\gamma_{\perp}$ ) and  $\psi$  is the laser-empty cavity detuning.

Figure 1 exhibits the full complexity of the intracavity dynamics represented by Eq. (1). Co-existing attractors appear on the low transmission bistable branch. Furthermore, the shaded domain encloses a stable six cycle and its associated subharmonic bifurcations. It is evident from this picture that Eq. (1) when viewed as a mapping of the complex plane to itself contains multiple basins of attraction associated with different attractors. These behaviors appear more consistent with the two dimensional Henon map rather than Feigenbaum's one dimensional logistic map.

When transverse variations are allowed for new transition sequences appear which are consistent with the Ruelle-Takens picture of the transition to turbulence.<sup>4</sup> Bifurcation sequences involving outputs which go through periodic, quasiperiodic, frequency locked, followed by period doubling of the locked outputs are common, in marked contrast to plane wave predictions (see Figure 2). The numerically computed dynamical outputs of the optical cavity show remarkable similarities to measured radial velocity outputs from a recent fluid dynamical experiment.<sup>5</sup> These latter results emphasize the important role played by the Fresnel number in determining the nature of the transition to optical turbulence.

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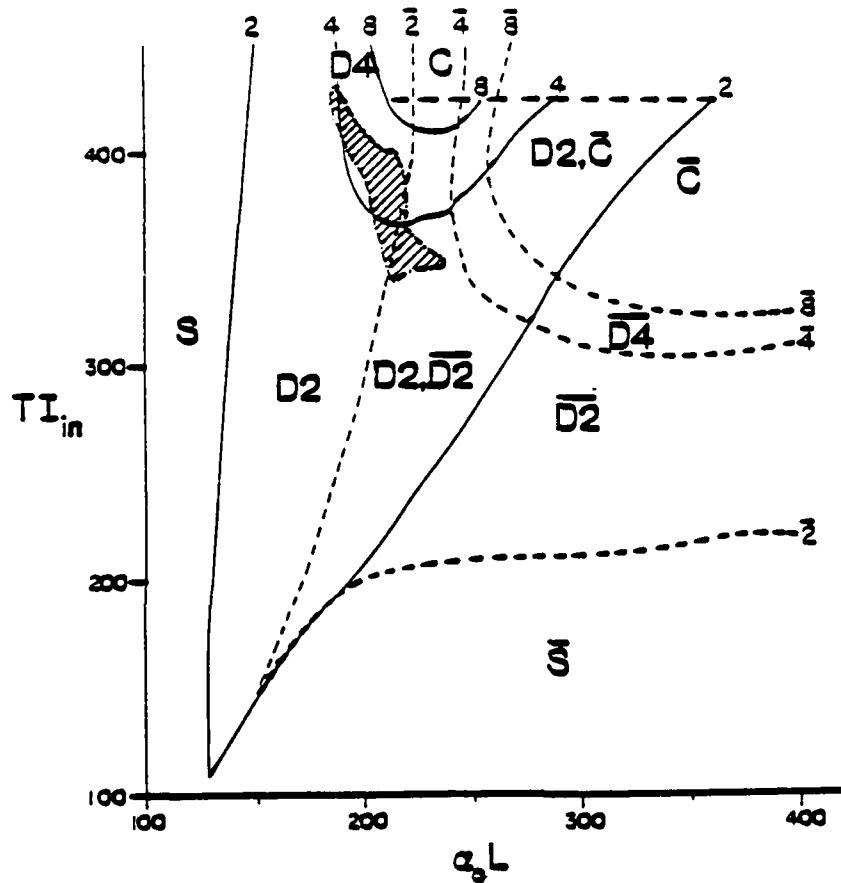


Figure 1. Bifurcation diagram for Eq. (1) in  $(\alpha_0 L, TI_{in})$  parameter space. Fixed points in  $S$  and  $\bar{S}$  bifurcate to the respective chaotic attractions  $C$  and  $\bar{C}$ . The crosshatched domain encloses a stable six cycle which appears to arise by a tangent bifurcation. All three bifurcating attractors coexist in some regions of parameter space.

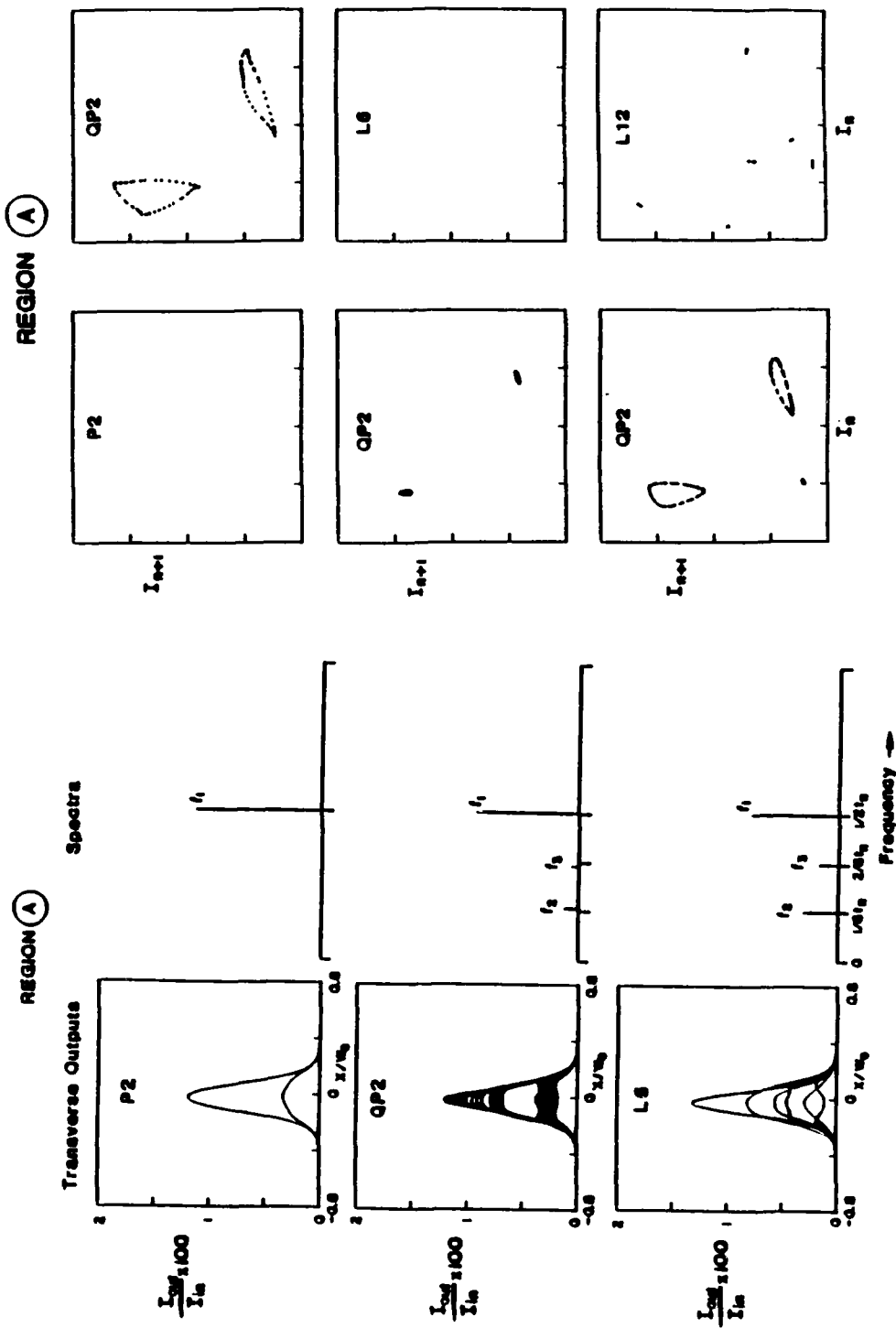


Figure 2. (a) Transverse outputs on successive cavity roundtrips. (b) Frequency spectra (schematic) of the on-axis dynamic outputs showing frequency locking phenomenon and (c) Poincaré surface of section. This sequence shows a Hopf bifurcation from a stable limit cycle to motion on a torus followed by a breakdown of the torus (Ref. 4).

Resonant Frustrated Total Reflection (FTR) Approach to  
Optical Bistability in Semiconductors.

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SUMMARY

The recent observation of very large non-linearities and of optical bistability in semiconductor structures at room temperature has heightened the interest in all-optical switching and signal processing as a practical prospect in optical information technology. Along with the search for suitable non-linear materials, some attention is also being paid to the study of structural configurations designed to optimize the performance of a bistable device. In this paper we will discuss the resonant FTR (Frustrated Total Reflection) approach, as an experimental configuration fully compatible with semiconductor and integrated optics technologies, which should lower the power requirements of a bistable device through the "enhancement" of the optical field inside its non-linear region.

At the basis of the resonant FTR approach is the concept of non-linear FTR optical cavity, a thin film structure into which light is coupled by total internal reflection, and whose cavity layer is made of a non-linear medium. (Fig.1) At resonance, a very intense electric field builds up in the cavity layer (enhancement factors of 2 to 3 order of magnitude are common), and a sharp minimum occurs at  $\theta_m$  in the angular reflectivity spectrum of the

structure.  $\theta_m$  depends on the refractive index  $n_2$  of the cavity layer ( $n_2 = n_{20} + \beta |E_2|^2$ , where  $n_{20}$  is the refractive index at low intensity,  $\beta$  the non-linear Kerr coefficient, and  $|E_2|^2$  the optical field intensity inside the cavity layer). Conversely, at a fixed angle of incidence  $\theta_i$ , appropriately chosen,  $|E_2|^2$  depends on  $\theta_m$ , being maximum when  $\theta_m = \theta_i$ . From the simultaneous and reciprocal interdependence of  $|E_2|^2$  and  $n_2$ , optical bistability is expected to occur.

We have performed a numerical study of the optical response of a non-linear FTR optical cavity as a function of the incident power, using the plane wave approximation and assuming a uniform enhancement of the optical field in the cavity layer. A typical result is shown in Fig.2. Preliminary estimates of the critical intensity  $I_1$  for the switching of the system from high to low reflectivity regime suggest that, with respect to the non-linear interface configuration, a reduction of at least one order of magnitude can be expected. This fact, and the possibility of a convenient angle of incidence, point to the non-linear FTR optical cavity as a better candidate for the experimental study of the non-linear effects predicted by Kaplan.<sup>1</sup> We will also discuss the relationship, and some potential advantages, of the resonant FTR approach with respect to the waveguide and the surface-plasmon configurations.

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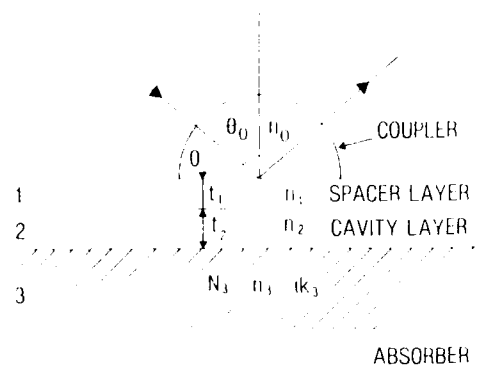


Fig.1 - Schematic representation of the FTR optical cavity.

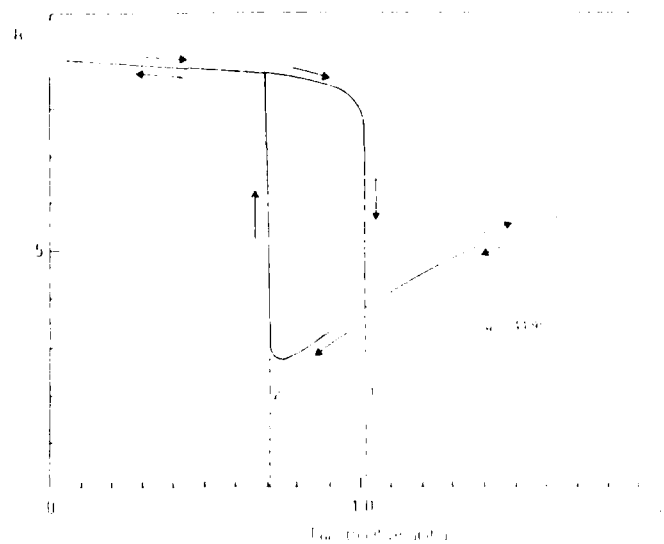


Fig.2 - Typical 'reflectivity vs. incident intensity' response of an FTR optical cavity.

Optical hysteresis and bistability in the  
wave-front conjugation by electrooptic  
crystals

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Summary

Analitical and numerical investigation of the wave-front conjugation in electrooptic crystals ( $\text{LiNbO}_3$ ,  $\text{BaTiO}_3$ , SBN) show that moncavity optical bistability and hysteresis /1,2/ depends on the dominant mechanism of nonlinear response (diffusion or drift) as on the orientation of the crystal C axis, and is strongly influenced by absorption or gain. Calculation show that critical effects may be observed for crystal length 0.1 cm (SBN,  $\text{BaTiO}_3$ ) and 1.5 cm ( $\text{LiNbO}_3$ ) for the optimal experimental geometry.

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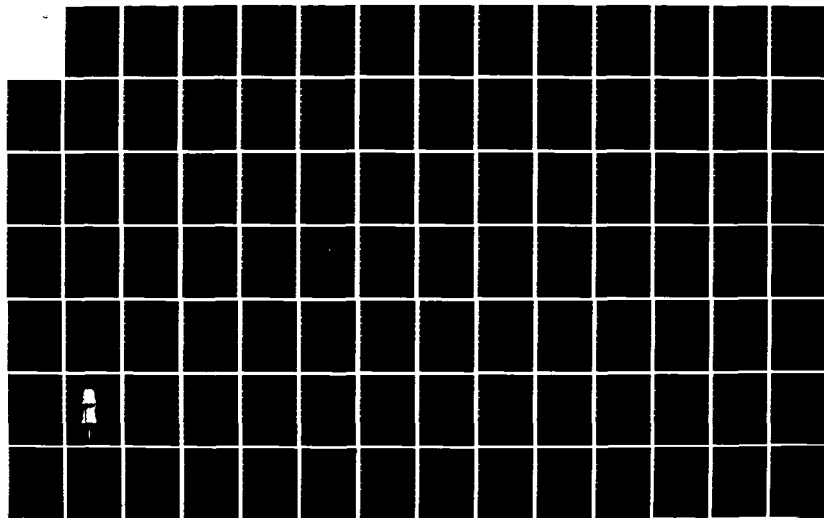
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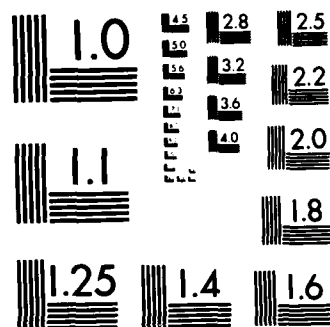
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## Observation of Bistability in a Josephson Device

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We have been building devices with two Josephson junctions coupled together by a transmission line or waveguide. We have found that these devices can exhibit a type of bistable behavior not seen before in other Josephson devices. The observed behavior appears to have certain similarities to optical multistability.

The major observation is that the output variable of this device (a voltage) will switch spontaneously back and forth between two values that are in the ratio of roughly 5:2. This is qualitatively different from the type of bistability conventionally observed in Josephson devices: there one always finds that one of the states is the zero-voltage state.

Our hypothesis is that these two (nearly?) stable states represent the phase-locking of the oscillations of the Josephson junctions to either one or the other of the two lowest order odd half-wave resonances of the transmission line. This would then explain the two output voltage states, because the Josephson relation requires that the average frequency and the voltage be in exact proportion.

We actually have not yet observed true stability of the two

states. The spontaneous switching between states has not yet been observed to cease. The average switching rate has been observed to vary systematically, as a function of the two input parameters, over a range of  $< 10^2$  to  $> 10^5$  per second. This is much slower than the Josephson oscillation frequencies characteristic of the device ( $10^{10}$  to  $10^{11}$  Hz). At this time we can not say if the cause of the switching is intrinsic, or if it is induced by external perturbations.

## Theory of Multistability in Josephson Junction Devices

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The existence of phase-transition like behavior in Josephson junctions is well known. However most studies of the theory of these transitions are restricted to numerical calculations, owing to the nonlinearities of the Josephson equations in the regions of usual interest. Approximate analytic techniques that do exist are often restricted to regions where the oscillations are nearly sinusoidal.

The theoretical technique introduced here is, instead, a nonlinear perturbation theory that includes the nonlinearities of the Josephson equations to zeroth order. A renormalization of the zeroth order solution is then used to remove secular terms from the perturbative solutions. This results in relatively simple input-output relations that are applicable to typical current-driven Josephson devices.

The renormalization method is first used to deal with the theory of the onset of capacitive (lightly damped) bistability in single junctions, in general agreement with previous work. Next the theory of waveguide superconducting quantum interference devices is described. Optical multistability is obtained in the

sense of multistable mode intensities in the waveguide. The state-equations depend on the external flux, through the flux-quantization relations. Either even or odd order modes can become excited provided the waveguide can support the relevant mode.

A physical picture of the origin of the multistability is presented, and it is predicted that the upward transitions (i.e., increasing frequencies) should have threshold frequencies ( $\omega$ ) and currents ( $I$ ) related by

$$(I/I_c)^2 = 1 + (\omega T)^2$$

where  $I_c$  is the critical current and  $T$  is the junction characteristic time scale.

The fine scale time evolution of the system was simulated numerically. The initial conditions were varied to determine where the equations had more than one solution. The analytic results for the input-output relations are compared to these simulations.

It is noted that optically multistable Josephson devices have similar potential switching rates to capacitive (lightly damped) bistable junction devices. The types of junction transitions described here could therefore have applications in switching devices.

Finally, while the odd order multistability results appear to approximately agree with input-output relations that we have observed in the laboratory, at least one part of the theory presented here has not yet been tested in experiment. This is the prediction of even order mode excitations. In order to obtain

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these, one requires a type of waveguide able to support symmetric modes. This therefore leaves an open area for experiment which should provide a stringent test of the theory.

## Intrinsic Instabilities in Homogeneously Broadened Lasers

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Experimental observation has shown that it is no simple matter to make a homogeneously broadened laser operate in a single longitudinal mode. However, elementary theoretical treatments of homogeneously broadening predict that such lasers can operate in only a single mode, because all of the atoms constituting the gain medium experience identical degrees of saturation.

These same arguments apply in the case of a homogeneously broadened absorber. However, we have recently demonstrated that the absorption profile as seen by a weak probe field is not uniformly saturated.<sup>1</sup> We have saturated the green absorption band of ruby with an argon ion laser and measured a pronounced dip in the absorption near the argon laser line. The dip had a width  $T_1^{-1}$  of 37 Hz (HWHM).

The measured absorption profile was in good agreement with a previously published theory.<sup>2</sup> This theory shows that the saturated medium is unstable to the development of self-modulation which allows modes symmetrically displaced from the original mode to grow. These modes are locked in a definite relative phase and amplitude. They are natural modes which satisfy a simple eigenvalue equation.

In this paper we will show that this effect occurs in an

amplifier as well as in an absorber, and that in the case of an amplifier it can lead to higher gain in the wings than near line center. When the laser is operating in a single mode well above threshold the gain at approximately one Rabi frequency on either side of the lasing mode can exceed threshold. Thus the laser becomes unstable and switches to multimode behavior with a characteristic mode separation.

We will apply this theory to the case of a cw rhodamine 6G unidirectional ring dye laser pumped by an argon ion laser, and will present the results of some experiments showing that this system demonstrates the instability.

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## **Zeeman Coherence Effects in Absorptive Polarization Bistability**

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The phenomenon of optical bistability is usually treated within the framework of a single optical mode interacting with a two-level system. Recently, a number of authors<sup>1-5</sup> have considered the two-mode case where the mode coupling gives rise to new interesting features such as optical tristability.<sup>2</sup> In a specific case of particular interest, the coupling between orthogonally circular polarized modes arises from a two-photon resonance of the Zeeman sublevels.<sup>2,5</sup> Bistability or tristability is then observed in the polarization characteristics of a single optical beam. Although Zeeman coherence effects are expected to play an important role, these have usually been neglected in a theoretical description of polarization bistability.<sup>2,4</sup>

In this paper we consider the effect of Zeeman coherence on absorptive polarization bistability. The nonlinear medium placed inside a ring cavity is modeled in terms of a homogeneously broadened, folded three-level system. Its response to an arbitrarily polarized incident beam is obtained using the density-matrix formalism under fairly general relaxation conditions. In particular, the collisional decay of Zeeman coherence is allowed through a dephasing relaxation rate. The results indicate that the mode coupling decreases with the increase in the Zeeman-coherence dephasing rate.

In the absence of collisional dephasing, the bistability state equation gets simplified considerably. The situation is similar to that of a two-level system except for that both modes participate in optical saturation of each one-photon resonance.<sup>3</sup> Novel features arising from this cross saturation are discussed and analyzed. The collisional decay of Zeeman coherence is shown to affect substantially the behavior of such an optical bistable device. The attention is also paid to the effects of Zeeman splitting in the presence of an axial magnetic field.

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## Effect of Driving Laser Fluctuations on Optical Bistability

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We derive and analyze the master equation for absorptive and dispersive optical bistability in the high-Q cavity limit where the driving laser has both amplitude and phase fluctuations which are larger than the small amplitude and phase fluctuations caused by incoherent spontaneous emission in the cavity. We treat the amplitude and phase fluctuations of the driving laser as independent time dependent Gaussian processes. The causes of jitter and amplitude fluctuations will vary from laser to laser and the fluctuations need not be Gaussian. However, we use Gaussian processes for mathematical tractability and because we expect them to be qualitatively correct. Recently,<sup>1</sup> we have shown in the case of absorptive optical bistability when the driving laser has phase fluctuations the fundamental process which determines the statistical properties of the internal field is the competition between locking of the internal field phase to the phase of the driving laser and the diffusion of the phase of the internal field as a consequence of the driving laser fluctuations. The driving laser fluctuations have little effect if the phase locking rate is larger than the phase diffusion rate caused by the driving laser's fluctuations. However, when the diffusion rate becomes comparable to the locking rate we find the amplitude of the internal field is reduced and the dimensionless parameter<sup>2</sup> measuring the importance of transitions from one branch of the OB curve to the other due to fluctuations takes on a value which leads to an appreciable transition rate due to fluctuations.

In this paper we generalize our previous work to include the effect of driving laser fluctuations on the amplitude of the internal field. The amplitude of the internal field is more sensitive to driving laser fluctuations than is the phase of the internal field because the internal field amplitude exhibits critical slowing down at the turning points of the OB curve while the internal field phase variable does not. We have found that the effect of amplitude and phase fluctuations of the driving laser have considerably different consequences for absorptive and dispersive OB. For example, if the driving laser has only phase jitter then in absorptive OB only the phase of the internal field is directly affected by the jitter. On the other hand, in dispersive OB the driving laser phase jitter affects directly only the amplitude of the internal field variable. If the driving laser has both amplitude and phase fluctuations, then the fluctuations of the internal field variables depend on two ratios: (1) the relative strength of the driving laser's amplitude and phase fluctuations and (2) the relative magnitude of the dispersion and absorption. In order to derive the master equation we start with the Maxwell Bloch equations with a fixed driving laser field and then we replace the deterministic driving laser field by a stochastic driving field which has both amplitude and phase fluctuations. If the correlation times of the driving laser fluctuations are short enough to satisfy the Kubo condition,<sup>3</sup> we can use the Stratonovich procedure and obtain a Fokker-Planck equation in which the independent variables are the matter polarizations, matter inversion and internal field variables. Next we take the high-Q cavity limit which allows us to eliminate the matter variables adiabatically. The resultant Fokker-Planck equation has nonlinear diffusion coefficients which depend

on the relative phase of the driving laser and the internal field and on the driving laser amplitude. The fluctuations of the driving laser also modify the Fokker-Planck drift terms by terms that can be appreciable. We solve the linearized master equation and compute the spectra. We also investigate the spectra in the case where the correlation time of the driving laser fluctuations is not sufficiently short to justify the derivation of a Fokker-Planck equation.

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Optical Bistability in Nonlinear Media:  
An Exact Method of Calculation

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In a recent paper<sup>1</sup> I formulated a method for calculating propagation through nonlinear media for a number of nonlinear optical phenomena. This method does not use the SVEA and can therefore solve nonlinear propagation problems when refraction and reflection at the boundary interfaces are important. Here, I report the first calculation using this exact (albeit numerical) method. For this first test of the utility of the method, I have chosen to study optical bistability in a thin film whose index of refraction is intensity dependent.

Most treatments of bistability describe the phenomena by writing down the transmission of a Fabry-Perot interferometer,  $\tau$ , as a function of the intracavity phase shift in a roundtrip pass,  $\delta$ :  $\tau = (1 + F \sin^2 \delta/2)^{-1}$  where  $F = 4r/(1 - r^2)$  and  $r$  is the reflectivity from the surface of the material.<sup>2</sup> When the index of refraction is a function of the intensity inside the cavity ( $I_{in}$ ), the phase shift is intensity dependent, and it appears that one can substitute the expression for the intensity dependent phase shift into the expression for the transmission thereby obtaining an expression for the transmission as a function of the intensity inside the cavity. When this equation is solved in conjunction with an expression for the transmission as a function of the ratio of  $I_{in}$  to the incident intensity ( $I_0$ ) one obtains an expression for  $\tau(I_0)$  which may be multivalued. The flaw in this argument is that the expression for the transmission as a function of phase shift is derived assuming linear wave propagation where the super-

position principle applies.<sup>2</sup> But the propagation equation is nonlinear, and the superposition principle is not valid!

In Ref. 1, the exact propagation equations for the reflection and transmission coefficients ( $R$ ,  $T$ ) for a plane wave with field strength,  $|E_0|^2 = y$ , incident normally on a slab of material of length  $L$  with an intensity dependent refractive index. Using the same notation, as in Section II of Ref. 1, the nonlinear wave equation inside the slab ( $Q \in (0, L)$ ) takes the form

$$(d^2/dQ^2 + k^2)E(Q) = V(Q, |E(Q)|^2)E(Q), \quad (1)$$

and the boundary conditions can be written as<sup>6</sup>

$$E(Q) = y[e^{-ikQ} + R(L, y)e^{+ikQ}], \quad Q > L \quad (2)$$

$$E(Q) = y T(L, y)e^{-ikQ}, \quad Q < 0 \quad (3)$$

where (in MKS units)  $k$  is the wavevector in vacuum,  $k^2 = \epsilon_0 \mu_0 \omega^2$ , and  $V$  is given in terms of the susceptibility,  $\chi$ , by

$$V(Q, |E(Q)|^2) = -k^2 \chi(Q, |E(Q)|^2). \quad (4)$$

The propagation equations for the reflection and transmission coefficients as a function of varying slab thickness,  $x$ , can be derived using the method of invariant inbedding. They take the form

$$\tilde{R}_x(x, y) = (2ik - iV/k) \tilde{R}(x, y) - iV/(2k) (1 + \tilde{R}^2(x, y)) + Vy/k \tilde{R}_y(x, y) \operatorname{Im} \tilde{R}(x, y), \quad (5)$$

$$T_x(x, y) = -iV/(2k) T(x, y) (1 + \tilde{R}(x, y)) + Vy/k T_y(x, y) \operatorname{Im} \tilde{R}(x, y), \quad (6)$$

where

$$\tilde{R}(x, y) \equiv e^{2ikx} R(x, y) \quad (7)$$

and

$$V \equiv V(x, |E(x, y)|^2) = V(x, y |e^{-ikx} + R(x, y)e^{ikx}|^2) = V(x, y |1 + \tilde{R}(x, y)|^2). \quad (8)$$

The initial conditions for  $\tilde{R}$  and  $T$  are

$$\tilde{R}(0, y) = 0, \quad (9)$$

$$T(0, y) = 1. \quad (10)$$

For finite  $y$  one can use the method of lines<sup>3</sup> to solve the partial differential equations in (5-6) together with the boundary conditions (9-10) and initial conditions for  $\tilde{R}(x,0)$ ,  $T(x,0)$ . The  $x$  coordinate is discretized and the solution is expanded in cubic Hermite polynomials in each subinterval so that a system of ordinary differential equations in the  $y$  variable is obtained. The system of ordinary differential equations is solved by a variable stepsize GEAR method. I used a model susceptibility for the nonlinear material InSb proposed by Weaire et al<sup>4</sup> wherein  $\chi(|E(Q)|^2) = n_1^2 - 1 + n_1 n_2 \epsilon_0 / \mu_0 |E(Q)|^2$  with  $n_1 = 4$ , and  $n_2 = -6 \times 10^{-9} \text{ m}^2/\text{W}$  at  $1886 \text{ cm}^{-1}$  ( $5.3 \mu\text{m}$ ) radiation. However, the method can be used for any (complex) susceptibility which is a function of  $|E(Q)|^2$ . Fig. 1 shows the reflectivity ( $|\tilde{R}|^2$ ) vs. intensity in films of thickness  $0.18$  and  $0.09 \mu\text{m}$  (a small fraction of a wavelength) where the incident radiation intensity was increased to  $260 \text{ MW/m}^2$  and then lowered back to zero.

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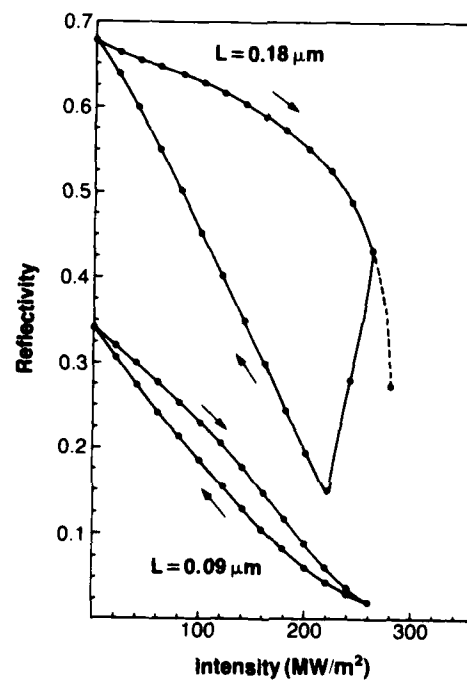


Figure 1 - Reflectivity vs. incident intensity in slabs of InSb of thickness 0.18  $\mu\text{m}$  and 0.09  $\mu\text{m}$ . Maximum intensity is 260  $\text{MW/m}^2$ . The dashed line of top curve indicates reflectivity if one increases the intensity to 280  $\text{MW/m}^2$ .



SELF-BEATING INSTABILITIES IN BISTABLE DEVICES

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Since the observation of optical bistability, several papers have discussed the effect of a delayed feedback on instabilities pointing out that the transmitted light from the system can be periodic or chaotic in time under certain conditions. This phenomenon was confirmed by Gibbs et al.<sup>1</sup> in hybrid bistable optical devices with PLZT as electrooptical material. Ukada et al.<sup>2</sup> have reported similar effects in  $\text{LiNbO}_3$ . In this case a coaxial line was employed as delay. A sustained oscillatory optical output is shown to appear under the unstable condition. Moreover, as we have shown previously,<sup>3</sup> similar effects can be obtained from liquid crystal cells under suitable conditions. In this paper we extend these results giving an analytical model of some of the obtained oscillatory optical outputs.

The schematic arrangement of our electrooptic bistable device is similar to the previously reported by us<sup>3</sup>. A twisted nematic cell, forming an angle of around  $45^\circ$  with respect to the input laser beam, is driven by a biasing voltage and a feedback voltage, which is proportional to the optical output power. With this arrangement, three are the time constants involved. The first one,  $T_1$ , is due to the photodetector. The second one,  $T_2$ , comes mainly from

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the capacitive effect due to the liquid crystal cell. The last one,  $\tau$ , is due to the time needed by the nematic molecules to reach the final position according to the applied voltage. Under these conditions, the differential equation governing the system is

$$V + (T_1 + T_2 + \tau) \frac{dV}{dt} + (\tau(T_1 + T_2) + T_1 T_2) \frac{d^2 V}{dt^2} + T_1 T_2 \tau \frac{d^3 V}{dt^3} = \beta I_W T(V + V_B)$$

where  $V$  is the feedback voltage,  $\beta$  is the feedback factor and  $T(V)$  is the cell transmission as a function of the voltage. As it can be shown, the system will oscillate with different frequencies depending on the system parameters. No oscillation appears if any of the above three terms is zero.

This theory is in a very good agreement with the output frequencies obtained with our twisted nematic cell. For low feedback and a biasing voltage of 0 volts, the optical output signal, has a frequency of 20 Hz. Its shape can vary by either changing slightly  $V_B$  or  $\beta$ . A higher increasing in  $\beta$  gives rise to a new frequency, in our case 70 Hz. Under some condition, a self-beating is obtained with the system oscillating from one frequency to the other. The beating frequency varies between 0.5 and 1 Hz.

A 5 mW He-Ne laser was employed as input beam. The detection system was a TIL78 phototransistor.

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EMPIRICAL AND ANALYTICAL STUDY OF INSTABILITIES IN HYBRID  
OPTICAL BISTABLE SYSTEMS

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In this paper we analyze the instabilities of electrooptic bistable devices with delayed feedback, giving the instability condition for oscillation to appear. This condition will be given for the two types of curves more commonly used, namely  $I_{out}$  VS.  $I_{IN}$  and transmission versus voltage.

As it is well known from the work of Gibbs et al.<sup>1</sup> optical turbulence and periodic oscillations are easily seen with a hybrid optically bistable device with a delay in the feedback. This behaviour has been studied, both of theoretically and experimentally, by several groups and different types of differential equations describe their properties. In the case of hybrid optical bistable devices the curve transmission versus applied voltage is the most useful parameter to work with. If the transmission value is expanded in Taylor's series around the working point

$$T(V+V_B) \approx T(\bar{V}+V_B) + \left( \frac{dT}{dV} \right)_{\bar{V}+V_B} (V-\bar{V}) + \dots$$

where  $\bar{V}$  is the equilibrium value, we obtain

$$\mu + \tau \frac{d\mu}{dt} = \beta I_W \left( \frac{dT}{dV} \right)_{\bar{V}+V_B} \mu(t-t_R)$$

$$\mu = V - \bar{V}$$

that for  $t_R \gg \tau$  (see Fig.1) gives

$$\left(\frac{dT}{dV}\right) \leq -\frac{1}{\beta I_{IN}}$$

for the oscillation condition

This condition, in the  $I_{OUT}$  VS.  $I_{IN}$  curve is

$$0 \leq \frac{dI_{OUT}}{dI_{IN}} \leq \frac{T}{2}$$

This condition is very straightforward to obtain if the transmission curve is known (Fig 2.a), because the above expression corresponds to

$$0 \leq \tan \theta_2 \leq \frac{1}{2} \tan \theta_1$$

If we study the  $\beta I_{OUT}$  versus  $V_B$  curve, assuming  $\beta$  constant and  $V_B$  varying, the instability region corresponds to (Fig 2.b)

$$-1 \leq \frac{d(\beta I_{OUT})}{dV_B} \leq -\frac{1}{2}$$

This condition is very easy to achieve for any particular case. We have developed an electrooptical bistable device with a twisted nematic cell as nonlinear material. For a constant input laser beam several oscillations were obtained for different values of  $V_B$ , in our case a triangular varying voltage. Depending on the  $\beta$  value either optical bistability or periodic oscillations were obtained.

Because the new concept developed in this paper it is possible to obtain oscillations in any hybrid optical bistable device, if the above condition is satisfied. This new method to study oscillations can help to a better understanding of

optical bistability in hybrid devices. In our case, optical multistability and periodic oscillations (20 Hz and 200 Hz) were obtained.

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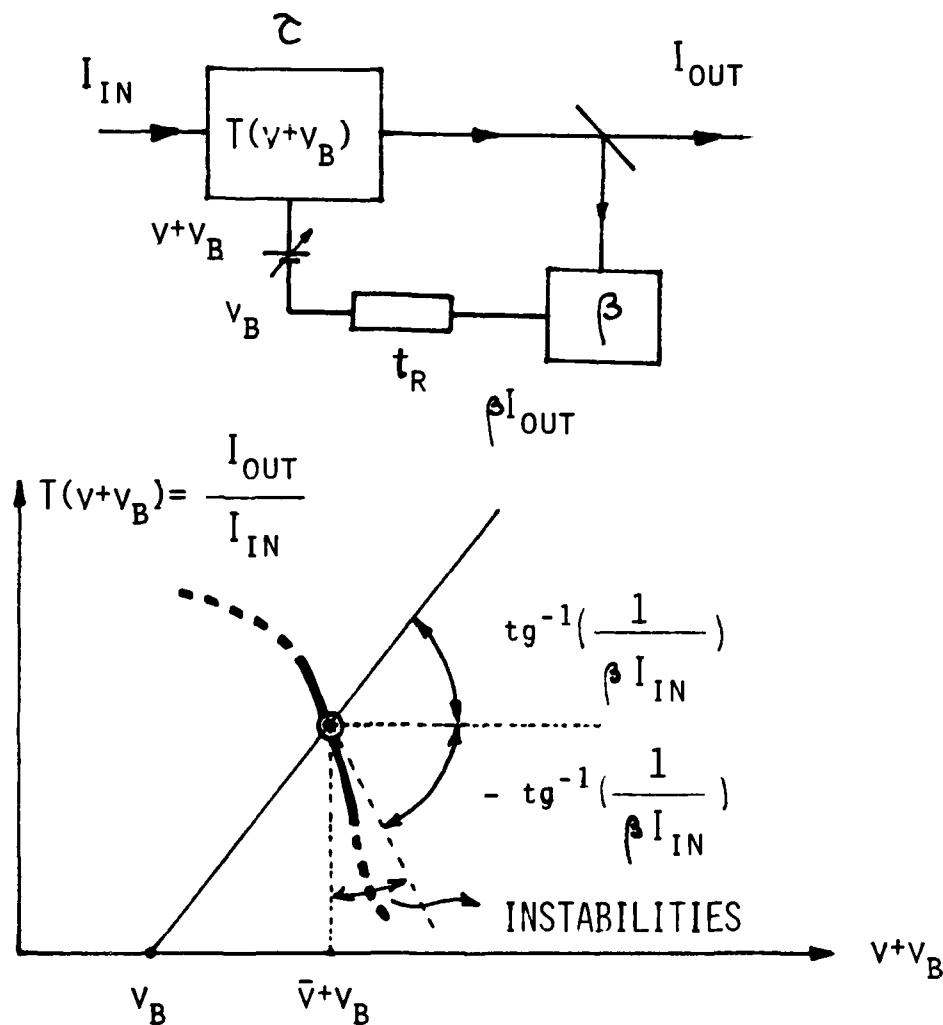


FIG. 1

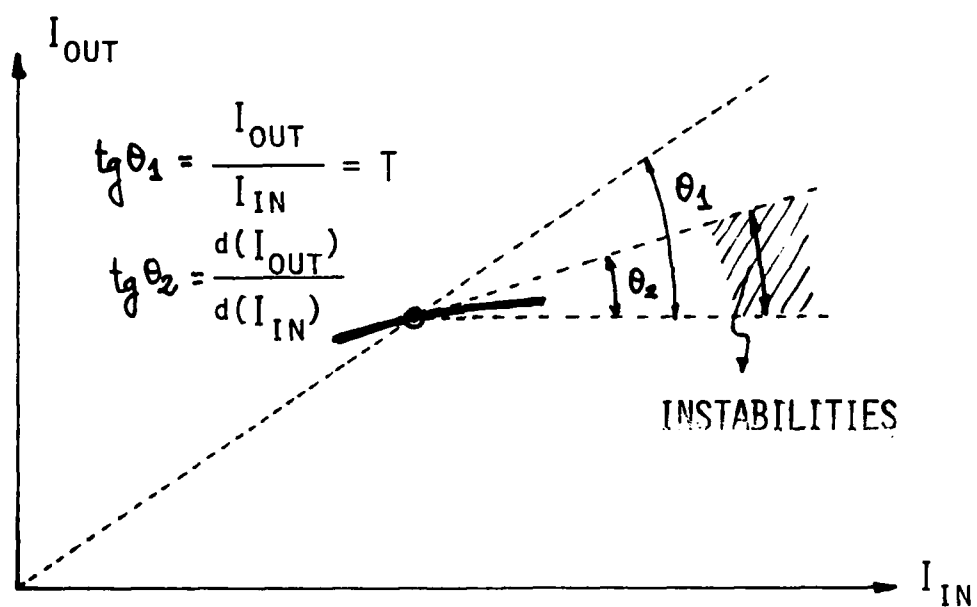


FIG. 2.a

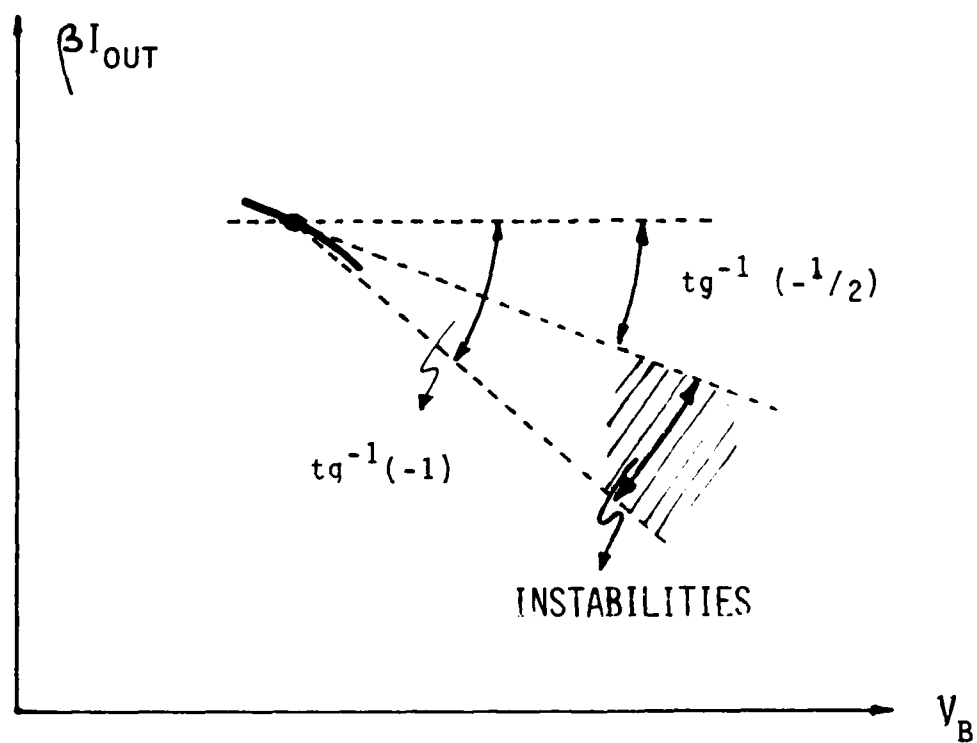


FIG. 2.b

## Distributed Feedback Bistability in Channel Waveguides

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Two current goals in the area of optical bistability are (1) to reduce the power required for switching between two states, and (2) to reduce the material volume (and hence power dissipation). An obvious approach is to minimize the cross-sectional area of the beam, for example by using channel waveguides of the type shown in Fig. 1. A grating milled into the surfaces can be used as the distributed feedback mechanism. In this paper we analyze a channel, guided wave, distributed feedback geometry and present numerical results for InSb waveguides.

Our method of analysis is a direct extension of previous work on distributed feedback bistability,<sup>1</sup> gratings for guided waves<sup>2</sup> and guided wave bistability.<sup>3</sup> The guided wave fields for both the rib structure and buried channel shown in Fig. 1 were calculated using the approximate formulae developed by Marcatili.<sup>4</sup> Using these field distributions, the intensity dependent propagation wavevector was calculated in the plane-wave approximation<sup>3</sup> which should be accurate to within  $\approx 10\%$  in the present case. Hence, in  $\Delta\beta = \beta_0 + \Delta\beta_0$  where  $\beta$  is propagation wavevector and  $P$  the power carried by the channel,

$$\Delta\beta_0 = \frac{k\epsilon_0}{2} \int_{\text{area channel}} n_0 n_{2,E} |E|^4 dydz$$



where

$n = n_0 + n_2 |E|^2$  is the intensity dependent refractive index and  $E$  is the guided wave field. The switching power is then simply given by  $P_s = 2/3 \Delta \beta_0 L$  where  $L$  is the interaction length along the channel.

The second required ingredient is the grating feedback parameter  $\Gamma$ . Usually, values in the range  $10 > \Gamma > 1$  are necessary for useful feedback. From coupled mode theory,

$$\Gamma = \frac{\epsilon_0 c k}{0} \int [n^2(z) - n_0^2] E_i \cdot t_r dz$$

where  $n^2(z)$  describes the modulation produced in the refractive index by the grating, and  $E_i$  and  $E_r$  are the incident and reflected guided waves respectively.

As an example, we chose InSb which exhibits very large nonlinearities<sup>5</sup> for  $\lambda \approx 5.5 \mu m$ . The modes of the channel are essentially TE in character in the sense that the largest component of the electric field is parallel to the surface and orthogonal to  $\beta$ . We assumed  $L = 1mm$ , gratings with a 20Å peak to peak ripple and sapphire as the substrate ( $n = 1.7$ ).

Typical results are shown in Fig. 2. They indicate that the power levels for switching can be as small as a few nanowatts in the optimum case. The ridge waveguide (1b) requires smaller powers because the beam confinement is optimum for that case. It also appears that gratings with amplitudes less than 100Å should be adequate for feedback purposes.

This research was supported by NSF (ECS-8117483), and ARO and AFOSR under the Joint Services Optics Program.

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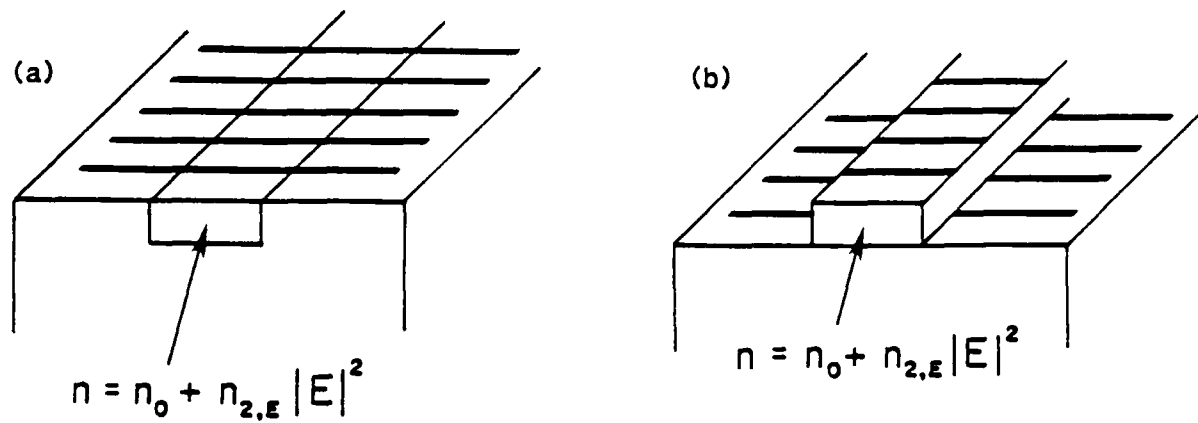


Figure 1. Buried (a) and ridge (b) channel waveguides with a surface grating on top.

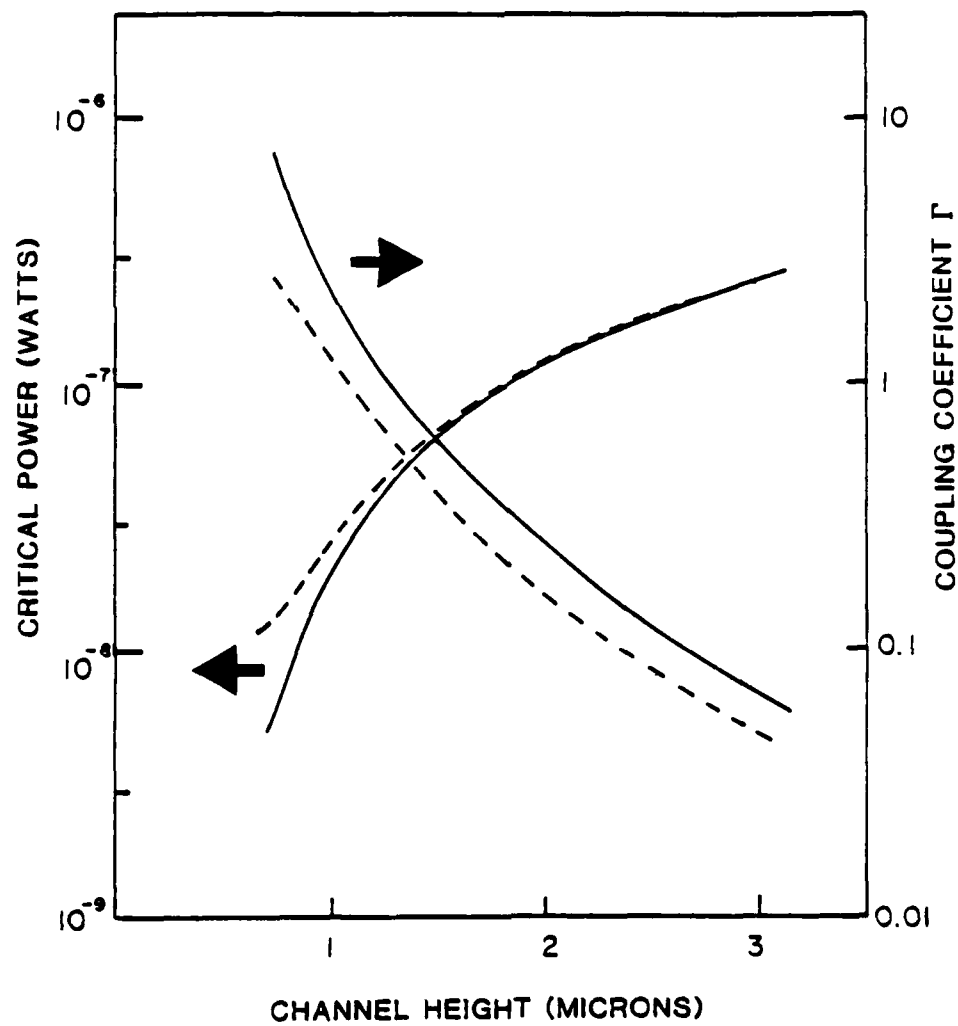


Figure 2. The critical power for switching and the coupling coefficient  $\Gamma$  for InSb channel waveguides with a height to width ratio of 0.4. The solid lines correspond to the ridge case and the dotted to the buried channel.

## Cyclotron Bistability of Electrons in Vacuum and Semiconductors

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The feasibility of an optical cyclotron bistability of free electrons in vacuum as well as of conductive electrons in semiconductors is discussed. It was recently shown by us<sup>1</sup> that the cyclotron bistability of the free electrons can be based on the dependence of a cyclotron frequency of forced oscillations on the relativistic mass of the electron and, hence, on its momentum (or kinetic energy). It was also suggested that<sup>1</sup> the analogous effect could be feasible in semiconductors whose conductive electrons have a particularly strong dependence of their effective masses on energy (or momentum) of their excitation. The main advantageous features of the cyclotron bistability in semiconductors is that (i) the effective mass of electrons in some semiconductors (e.g. InSb) is very small which results in a considerable decrease of a required magnet field for a given resonant frequency, and (ii) the nonlinearity associated with conductive electrons in semiconductors is by many orders of magnitude larger than the relativistic nonlinearity of free electrons, which allows for a fairly low critical pumping intensity, even if taking into consideration a faster relaxation time in semiconductors.

In this paper, the unified theory of both of those effects is developed. It is known that in both vacuum and narrow-gap semiconductors described by the Kane's two-band model<sup>2</sup> with anisotropic nonparabolic bands, the relativistic formula of the same kind for an effective mass  $\bar{m}$  of free ( $\bar{m} = m$ ) and conductive ( $\bar{m} = m^*$ ) electrons is valid:

$$\bar{m}/\bar{m}_0 = W/W_0 = \sqrt{1+p^2/p_0^2} = 1/\sqrt{1-v^2/v_0^2}. \quad (1)$$

Here  $\bar{m}_0$  - the rest mass (in semiconductors,  $\bar{m}_0 = m_0^*$  - effective mass at the bottom of a conductive band),  $W$  - kinetic energy,  $W_0$  - the rest energy ( $m_0 c^2$  - in vacuum,  $W_G/2$  - in semiconductors;  $W_G$  - the band gap);  $p$  and  $v$  - momentum and speed of the electron respectively;  $p_0 = \sqrt{W_0 \bar{m}_0}$  and  $v_0 = \sqrt{W_0/\bar{m}_0}$  - some scaling parameters ( $p_0 = m_0 c$ ,  $v_0 = c$  - in vacuum, and  $p_0 = \sqrt{W_G m_0^*/2}$ ,  $v_0 = \sqrt{W/2m_0^*}$  in semiconductors). In a classic approximation for the electron immersed in a constant magnet field  $H_0$  and subject to the action of a plane EM wave  $\vec{E}_{in}$ , the equation of the motion of the electron is:

$$\frac{d\vec{p}}{dt} + \Gamma \vec{p} = e \left\{ \vec{E}_{in} + \frac{1}{c} \nabla H_0 + \frac{\nabla}{\omega} [\vec{k} \vec{E}_{in}] - \vec{g} \right\}; \quad \nabla = \frac{\vec{p}}{\bar{m}(p)}; \quad (2)$$

where  $\Gamma$  - an inverse relaxation time of momentum,  $g$  - some trapping electrostatic field which is to cancel radiation pressure of EM wave represented by the term  $e\nabla[\vec{k} \vec{E}_{in}]/\omega$ . In vacuum, this field could be arranged by Penning trap, whereas in semiconductors, it is automatically generated by the redistribution of charges in a sample<sup>3</sup>;  $k$  and  $\omega$  - wave vector and frequency of the wave. Under weakly relativistic excitation (which is always valid under resonant conditions  $\Gamma \ll \omega_c$  and  $|\omega - \omega_c| \ll \omega_c$ ) and for a circular polarization of EM wave, the steady-state magnitude of the momentum  $p$  is determined by the equation:

$$(eE)^2 = p^2 \left[ \Gamma^2 + (\omega - \omega_c + \omega_c p^2 / 2 p_0^2)^2 \right], \quad (3)$$

where  $E$  - amplitude of the EM wave,  $\omega_c = eH_0/\bar{m}_0 c$  - an "unperturbed" (i.e. linear) cyclotron frequency. Under the critical conditions:

$$(eE)^2 = 16\Gamma^3 \bar{m}_0 W_0 / 3\sqrt{3}\omega_c; \quad \omega_c - \omega > \sqrt{3}\Gamma/\omega_c, \quad (4)$$

the steady-state momentum (3) becomes three-valued which results in both bistability

and hysteresis. In vacuum, if  $H_0 = 100\text{kG}$  ( $\lambda_c = 1.07\text{mm}$ ) and  $\Gamma/\omega_c = 10^{-4}$ , the critical incident power is  $\simeq 3\text{ W/cm}$  which can be readily achieved in cw regime. In semiconductor InSb,  $m_0^* = 0.014 m_0$ ,  $W_G = 0.24\text{ eV}$ , such that if  $\Gamma = 2 \times 10^{13}\text{ sec}^{-1}$ ,  $H_0 = 140\text{kG}$  (i.e.  $\lambda_c = 10.6\mu\text{m}$ , so that  $\text{CO}_2$  laser can be used as a source of exciting radiation), the critical incident power is  $\simeq 300\text{ W/cm}^2$  which again can be readily attained. In semiconductor, the light-induced electrostatic potential  $g$  exhibits bistability as well, which provides an immediate electronic indication of the intrinsic optical bistability.

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SWITCHING BEHAVIOR OF BISTABLE RESONATORS  
FILLED WITH TWO-LEVEL ATOMS

by

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A comprehensive numerical and analytical study of the switching dynamics of bistable optical devices is reported. The intracavity medium is modelled as an ensemble of two-level atoms which may be either homogeneously or inhomogeneously broadened. By allowing for arbitrary atomic detunings and cavity mistunings, both the absorptive and dispersive contributions to optical bistability are included. Standing waves are fully taken into account in the usual way through Fourier expansions.

The steady state analysis yields the very important result that even in the dispersive case when one is far detuned from the atomic resonance it is only possible to observe optical bistability if the intracavity intensity is of the order of the

saturation intensity or way above it. The implication is that even when operating in the dispersive mode, significant population of the excited state occurs. This population will not adiabatically follow the switching laser pulse, and thus switch-off of the device will occur on the same time scale as  $T_1$ , the population relaxation time, if, of course, we are in the bad cavity limit. Thus one does not necessarily gain in speed by operating under non-resonant conditions. This puts stringent restrictions for the operation of such a device as a memory element.

To confirm predictions based on steady state considerations, the time-dependent Maxwell-Bloch equations were solved numerically using the method of characteristics in finite difference form. In order to integrate the equations successfully over several  $T_1$  times, non-standard numerical techniques were required. Since rapid oscillations in the physical variables (such as polarization) lead to numerical instability, it was necessary to renormalize these variables with quasi-integrals of motion which vary more slowly. Details of the integration scheme will be presented. The numerical results show that  $T_1$  places a fundamental limit on signal processing speed in all-optical devices which require saturation of the absorption.

POLARIZATION BISTABILITY IN CHOLESTERIC LIQUID CRYSTALS

by

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Cholesteric liquid crystals possess an intrinsic helical structure which enables them to selectively reflect circularly polarized light if the wavelength of the light matches the pitch of the helix. We recently showed that intense light can change the pitch of the cholesteric and lead to a bistable reflection characteristic for wavelengths in the Bragg regime<sup>1</sup>. We have now extended our analysis to the regime of wavelengths much shorter than the helix pitch. In this limit (known as the Mauguin limit) there are no Bragg reflections and, except for normal dielectric reflections, an incident wave will enjoy 100% transmission. Since the cholesteric represents a twisted, birefringent medium, the polarization state of the light will evolve periodically in space.

The starting point of the analysis is the free-energy of the liquid crystal in the presence of the light field.



Minimization of the free-energy yields an Euler-Lagrange equation for the director field which describes the orientation of the elongated liquid crystal molecules. The light field couples to the local dielectric anisotropy to exert reorienting torques on the molecules whose orientation then affects the propagation of the wave. A simultaneous solution of the coupled Maxwell and Euler-Lagrange equations yields the self-consistent field and director distributions. An analytic solution is obtained by assuming the presence of only two co-propagating normal modes and by employing the slowly varying envelope approximation.

The results show that the polarization state of the transmitted light depends on the intensity of the input linearly polarized beam. Furthermore, for a range of incident intensities, there are two possible values for the ellipticity of the transmitted beam, and transitions between these two values are obtained. By placing the cholesteric liquid crystal between crossed polarizers it should be possible to observe an intensity bistability which does not rely on either Bragg reflections<sup>1</sup> or mirror feedback. The "nonlocal" character of the nonlinear interaction between light and liquid crystals will be shown to be important for this single pass, mirrorless bistability.

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# Simulations of the Dynamics of Optical Bistability with Fluctuations.

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The Langevin-equation (1) for the fluctuating light-field  $E$  in a nonlinear semiconductor resonator has been derived in Ref.[1]:

$$\frac{\partial E}{\partial \tau} = \kappa \epsilon_i - E(\kappa + \omega'' |E|^2) + E(\Delta + |E|^2) + \sqrt{Q/2} F(\tau) \quad . \quad (1)$$

All quantities are dimensionless,  $\epsilon_i$  being proportional to the amplitude of the incoming field,  $\kappa$  is a damping constant,  $\omega''$  is the ratio of imaginary and real part of the dielectric function and  $\Delta$  is proportional to the detuning. The additive term  $F(\tau)$  is the classical noise source representing a Gauss-Markov process. The stationary properties of eq.(1) have been investigated in Ref.[1] and the details of optical bistable operation has been determined. Here, we present numerical solutions of eq.(1) to study the dynamic effects of the system, such as the response to time-dependent incident fields  $I_0(\tau) = |\epsilon_i(\tau)|^2$ . We simulate the noise source  $F(\tau)$  using random numbers and obtain the most probable value of e.g. the field intensity in the resonator  $\langle I \rangle$  by averaging over many stochastically indepent realizations.

In Fig.(1) we show as a typical example the response to a linearly increasing incident field. Shown are the instantaneous quasistationary solution (dashed line — —) and the time-dependent solution (dashed line - - -) of the deterministic part of eq.(1).

Furthermore, we plotted a typical realization of the full stochastic equation (1) (thin full line) and the solution  $\langle I \rangle$  averaged over 1,000 realization (thick full line). The instantaneous solution jumps at  $\tau=32.5$  because at this time the end of the lower branch is reached. The time-dependent deterministic solution switches even later due to the inertia of the system. The average intensity  $\langle I \rangle$  shows the destabilization of the off-state due to the fluctuations. Already at  $\tau=25$  a certain percentage of realizations has switched to the upper branch (one example is shown in Fig.1) and the others follow, after around  $\tau \approx 30$   $\langle I \rangle$  reaches it's stationary value.

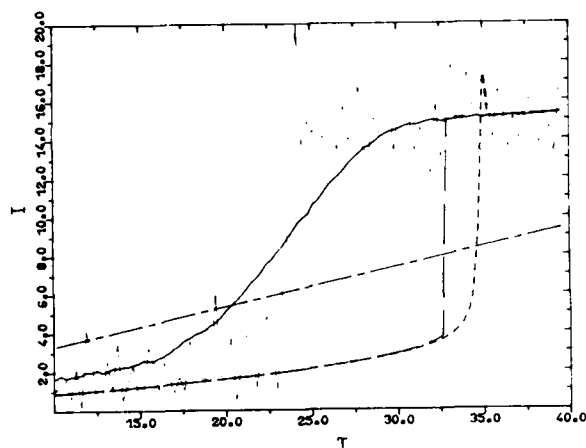


Fig.1: Response to linearly increasing  $I_0$  (see text). ( $I_0/100$  is shown as dashed line ---).

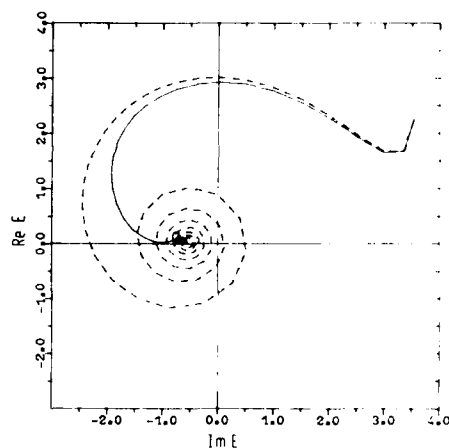


Fig.2: Response to instantaneous down-switching of  $I_0$  (see text).

The influence of the noise is also clearly visible in Fig.2. Here, we plotted  $E(\tau)$ ,  $E^*(\tau)$  as paths in the complex  $E$ -plane for an instantaneous switching of  $I_0$  from a value above to a value below the bistable region. The path parameter is the time. The deterministic solution of eq.(1) (dashed line in Fig.2) shows a

spiraling approach of the final state, a behaviour which has also been found in Ref.[2]. The noise however leads to a significant damping of this spiraling motion and the final state is reached directly (full line in Fig.2).

Furthermore, we investigated the stability of the on- and off-state against the noise in the bistable region. Our results can be fitted by an expression of the form

$$\langle I(\tau) \rangle = I_i e^{-\lambda \tau} + I_f (1 - e^{-\lambda \tau}) \quad , \quad (2)$$

where  $I_i$  and  $I_f$  are the initial and final value of the intensity in the resonator. The obtained decay rate of the on- and off-state is plotted in Fig.3 versus input intensity. As it is normally the case in optical bistability, we observe that the upper branch is more stable than the lower one due to the fact that the relative strength of the

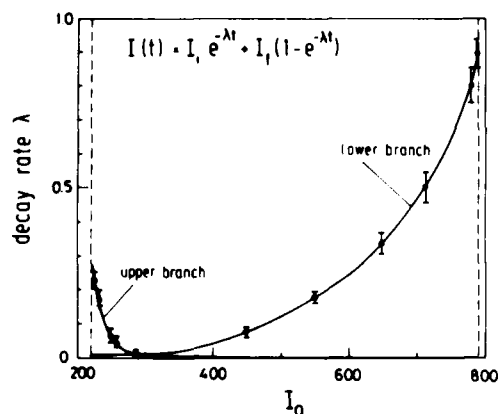


Fig.3: Decay rate  $\lambda$  versus input intensity.

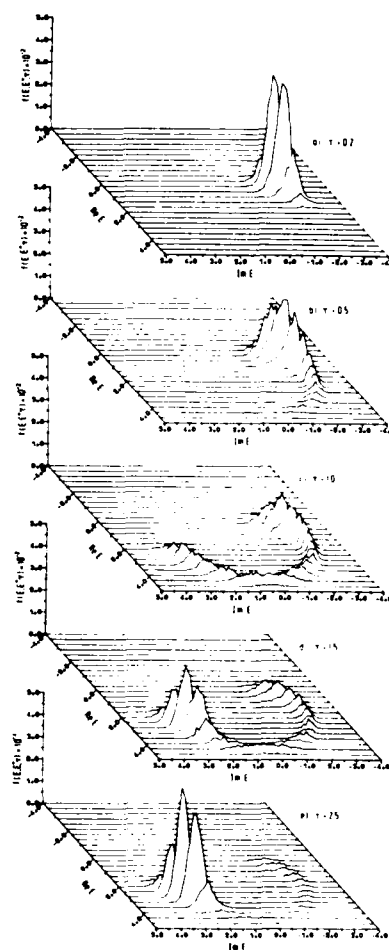


Fig.4: Probability distribution function  $f(E, E^*, \tau)$  for different times  $\tau$ .

fluctuations in the on-state is much weaker than in the off-state.

For one of these simulations, in which we watch the decay of the metastable off-state, we plot in Fig.4a-e the time-evolution of the full probability distribution function  $f(E, E^*, \tau)$  which has been obtained from 10,000 realizations of eq.(1). One sees the build-up and decay of the probability current over the potential barrier in the complex E-plane which separates the on- and off-states. Finally, the system has switched to the upper branch and approaches it's stationary distribution.

Summarizing our findings, we obtained the detailed time-dependence of optical bistability within the limitations of our model. The on-state shows a considerably higher stability than the off-state. Thus, the pronounced hysteresis effect predicted by the stationary deterministic solution is not fully destroyed by the external noise.

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NOTES

Instabilities and Chaos in TV-optical feedback

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1. About TV-optical feedback

The simplest TV-optical feedback experiment is performed by a TV-camera looking onto its own monitor. If we introduce some signal processing into the system as convolutions, image subtraction, nonlinearities, etc., there are many applications: in a first approximation we assumed linear systems and we obtained systems for image restoration, iterative image processing and for the solution of partial differential equations [1]. However, the world is nonlinear, and if we introduce nonlinearity in our feedback loops completely new effects may occur. These effects occurring in many different physical systems are called "synergetic" by H. Haken [2].

2. Instability in a system with saturation

In a linear feedback system all the normal modes evolve without mutual interaction: they decrease (or increase) exponentially in space/time [3,4]. If the system becomes nonlinear a few modes may suppress ("slave") all others. Because of this "mode slavery" we observe e.g. distinct

spatio-temporal oscillations in an unstable feedback system [3,4]. Very often these oscillations are started by some small fluctuations (noise), grow up exponentially and slave all other modes: thus a macroscopic structure (order) has evolved out of noise.

### 3. Bistable behaviour

If a saturation nonlinearity is iterated, we obtain a switching characteristic. This is the basic idea for the construction of flip-flop-arrays, which were reported previously [1].

### 4. Deterministic chaos

A classical example [5] for a dynamic system with chaotic behaviour is the iteration of the parabolic nonlinearity  $f(I) = \beta I(I-1)$ : if  $\beta$  exceeds a critical value the output is unpredictable in any practical sense, after few iterations. We introduced such a nonlinearity in a TV-optical feedback system identifying the "I" in the eq. above with the circulating intensity. Then the behaviour of the system depends on the coupling between adjacent image points. The result are (pseudo-)random patterns ("texture") whose spatial structure can be influenced by the spatial part of the operator in the feedback branch.

We will show experimental results, among them a movie on the spatio-temporal evolution of these effects.



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POLARIZATION SWITCHING WITH  $J=\frac{1}{2}$  TO  $J=\frac{1}{2}$  ATOMS IN A RING CAVITY

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This paper reports a steady-state theory for the polarization behaviour<sup>1</sup> of the output of a ring optical cavity that contains a vapour of atoms with a driven  $J=\frac{1}{2}$  to  $J=\frac{1}{2}$  transition. The input laser radiation may have arbitrary polarization and detuning. We have extended a previously reported theory<sup>2</sup> (which includes analytic results for zero detuning of the incident laser and zero magnetic field) to include detunings and magnetic fields.

The underlying physical mechanism in our model is a coupling (via optical pumping) between the  $\sigma^+$  and  $\sigma^-$  components of the radiation which provides a purely atomic feedback in addition to the usual cavity feedback responsible for optical bistability. This atomic feedback depends on the decay rates of the upper and lower level orientations, expressed (at zero transverse magnetic field) through a single parameter  $\beta$ . The value of  $\beta$  largely determines the behaviour of the system: when  $\beta > 2$  polarization switching, for example as illustrated in figure 1, is expected to occur while for  $\beta \leq 2$  ordinary optical bistability preempts polarization switching.

The phenomenology shown in figure 1 is the same as that predicted using a suitably chosen three-state atomic model<sup>3</sup>: as the intensity increases, the system reaches a bifurcation point at which one of the  $\sigma^\pm$  components of the output increases in intensity while the

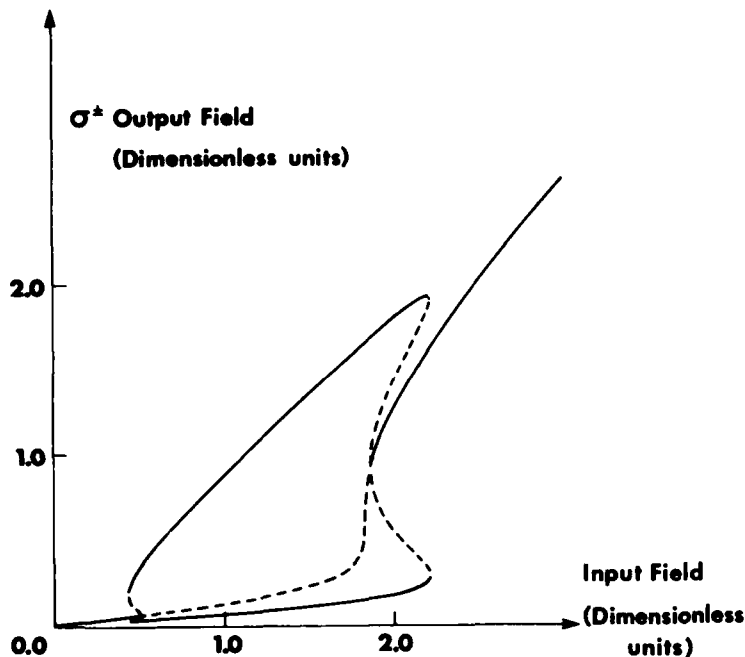


FIGURE 1. Polarization switching for  $\beta = 94$ .

other decreases. At still higher input intensity, the output reverts to linear polarization due to saturation of the transition.

We find that if we remove the symmetry between the  $\sigma^+$  and  $\sigma^-$  inputs, either by using elliptical input polarization or, for a detuned laser, by applying a magnetic field (along the laser propagation direction) the ambiguity in the switching of the output to predominantly  $\sigma^+$  or  $\sigma^-$  is removed. Otherwise, the phenomenology is similar to the case of linearly polarized input as we have shown by numerical solution of the state equation.

The application of a magnetic field transverse to the laser propagation direction<sup>4</sup> (for linear or nearly linear polarization) is shown to be a significant control parameter. We have analytically solved for the atomic polarization in steady state and found that the parameter  $\beta$  must be modified. For a sufficiently large transverse

magnetic field, the effective value of  $\beta$  will be small enough that polarization switching is suppressed and ordinary optical bistability restored.

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Optical Bistability Based on  
Self-Focusing

BY

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SUMMARY

Two years ago, the first proposal and demonstration of self-focusing optical bistability was published.<sup>1</sup> In this paper it was shown that a self-focused beam in a nonlinear medium would exhibit hysteresis if the beam were reflected back on itself. The effect was demonstrated using a dye laser beam and Na vapor as the nonlinear medium. The optical hysteresis observed is in good qualitative agreement with a recently - published analysis of this system.<sup>2</sup> By taking account of the saturation of the nonlinearity, more complex behavior has been predicted.<sup>2</sup> This behavior is due to oscillations of the spot size as the self-focused beam propagates through the nonlinear medium. In this paper we report on our experimental studies of optical multistability due to these spot size oscillations.

Experimental observations were made on a nonlinear system using a suspension of dielectric particles as the nonlinear medium.<sup>3</sup> The combination of large nonlinear coefficient (calculable from first principles) and slow response time makes this medium attractive for CW and transient studies of complex nonlinear effects. For these experiments, the medium was contained in a cell with an antireflection-coated front face, and with the interior surface of the exit window coated for 95 percent reflection. The light source was the 4880 Å output of an Ar<sup>+</sup> laser.

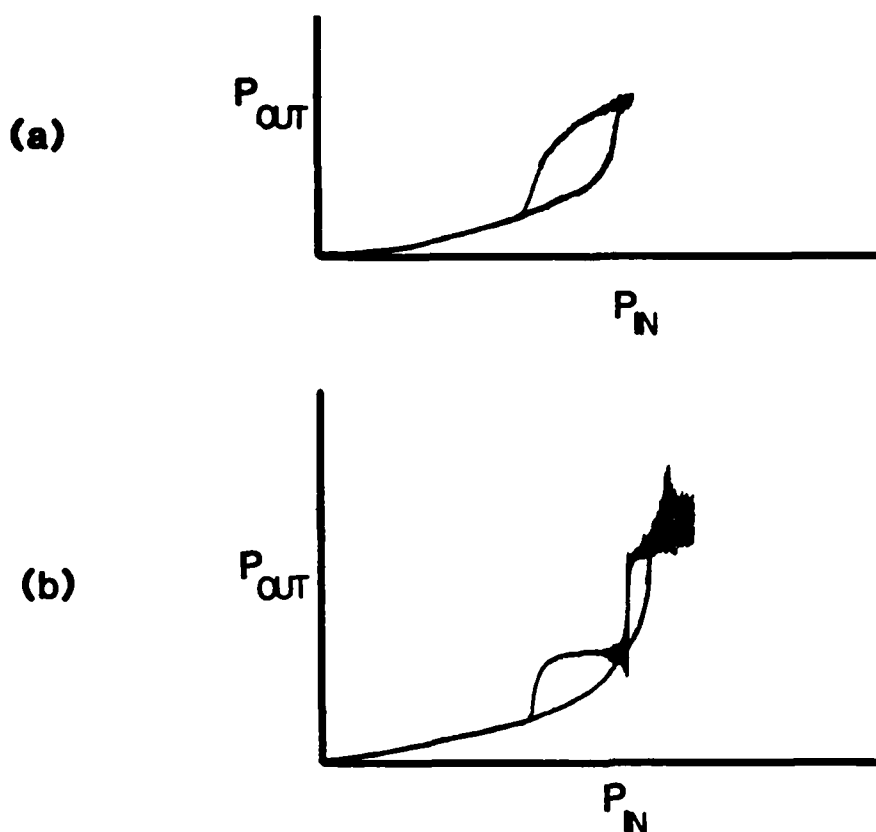


Figure 1 Experimental results

Figure 1 shows a sample of (a) hysteresis and (b) multistability due to oscillations in the self-focused beam size. These results will be compared with the predictions of the approximate theory of Reference 2.

These studies with model systems using dielectric suspensions as the nonlinear medium are useful for developing an understanding of their complex nonlinear behavior. Devices employing a fast-responding nonlinearity should prove attractive for subpicosecond switching elements.

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## OPTICAL BISTABILITY IN FOUR-LEVEL NONRADIATIVE DYES

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Most saturably absorbing dyes show a large unstaturable loss which removes the possibility of purely absorptive bistability. However, nonradiative dyes such as those used for Q-switching have an intensity-dependent phase shift which may allow bistability. We will show that this nonlinear refractive index comes from the population redistribution during saturable absorption.

Consider the four-level model shown in Figure 1, in which absorption occurs for both the  $1 \rightarrow 3$  and  $2 \rightarrow 4$  transitions. Lack of fluorescence implies that the  $3 \rightarrow 2$  and  $4 \rightarrow 2$  transitions are so fast that the steady-state populations in levels 3 and 4 are near zero. At intensities far above the ground-state saturable absorption intensity, most of the dye population is in the metastable state 2, and the steady-state susceptibility is given by  $\chi = N_2 \chi_2$ . Thus a change in population  $N_2$  causes an intensity-dependent phase shift. From rate equations we obtain

$$\chi = N p (1 + p)^{-1} \chi_2$$

where  $N$  is the total density of absorbers and  $p = I/I_s$ , the intensity inside the cavity normalized to the ground-state saturable absorption intensity.

To make numerical predictions, we relate the transmitted intensity  $P_t$  to the input intensity  $P_i$  in a Fabry-Perot cavity with mirrors of reflectivity  $R_1$  and  $R_2$  through the relation



$$P_i = P_t \frac{(1-R)^2}{(1-R_1)(1-R_2)e^{-\alpha L}} \left[ 1 + F \sin^2 \left( \frac{\beta + \Delta\phi}{2} \right) \right]$$

where  $\alpha$  is the unsaturated absorption,  $R = (R_1 R_2)^{1/2} e^{-\alpha L}$ ,  $F = 4R/(1-R)^2$ ,  $\beta$  is the detuning parameter of the Fabry-Perot cavity,  $L$  is the cavity length and  $\Delta\phi$  is the nonlinear phase shift.

This equation was numerically evaluated using experimentally measured parameters for BDN dye dissolved in dichlorethane. These parameters were determined by measurements of the intensity-dependent absorption for several dye concentrations. The results show that there is only a range of detunings for which the hysteresis curve is measurable. The optimum detuning is off resonance, indicating that nonlinear dispersion is the dominant mechanism for bistability. However, bistability disappears for detunings which are either too large or too small. This is different from the case of a pure nonresonant nonlinearity and occurs because the saturable absorption must provide a population redistribution to cause a nonlinear index.

Figure 2 shows how the switch-on and switch-off intensities depend on the initial optical density and the detuning parameter for  $R_1 = 0.9$  and  $R_2 = 0.95$ . Bistability occurs only in region (b). Region (c) occurs because for a given optical density there is a threshold in intensity below which bistability cannot be observed, for any detuning. Region (a) occurs because for large detunings, the hysteresis curve becomes an increasingly small fraction of the switching intensity. For a given initial optical density, both the detuning and switch intensity must be chosen

for operating within region (b) to observe bistability.

We made measurements of optical bistability in BDN dye using a Fabry-Perot cavity 1 mm long with mirrors of reflectivity 0.9 and 0.95. The laser was a Q-switched Nd:YAG with a pulsewidth of 20 nsec and a peak power of about 5 MW. Bistability was observed for certain dye concentrations and incident intensities by measuring a change in the time-dependent behavior of the output, as determined from the oscilloscope traces. Transfer curves, determined from point-by-point analysis, demonstrated switch-on behavior and hysteresis. In Fig. 2 we have marked with o's the dye concentrations and peak intensities for which bistability was observed and with +'s those values at which bistability was not observed. It can be seen that there is excellent agreement between the results of the theoretical model and the experimental results.

Zhen Fu Zhu is a visiting scholar on leave from Beijing Institute of Environmental Features, Beijing, The People's Republic of China. This work was sponsored in part by NSF.

Figure 1. Four-level model for saturable dye with unsaturated absorption loss.

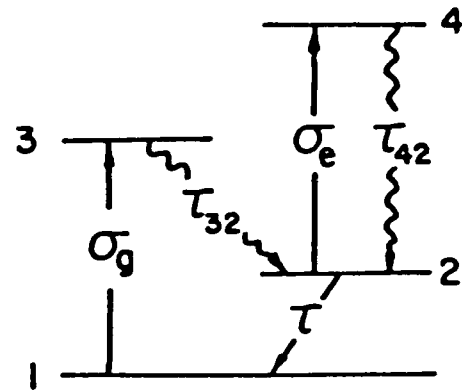
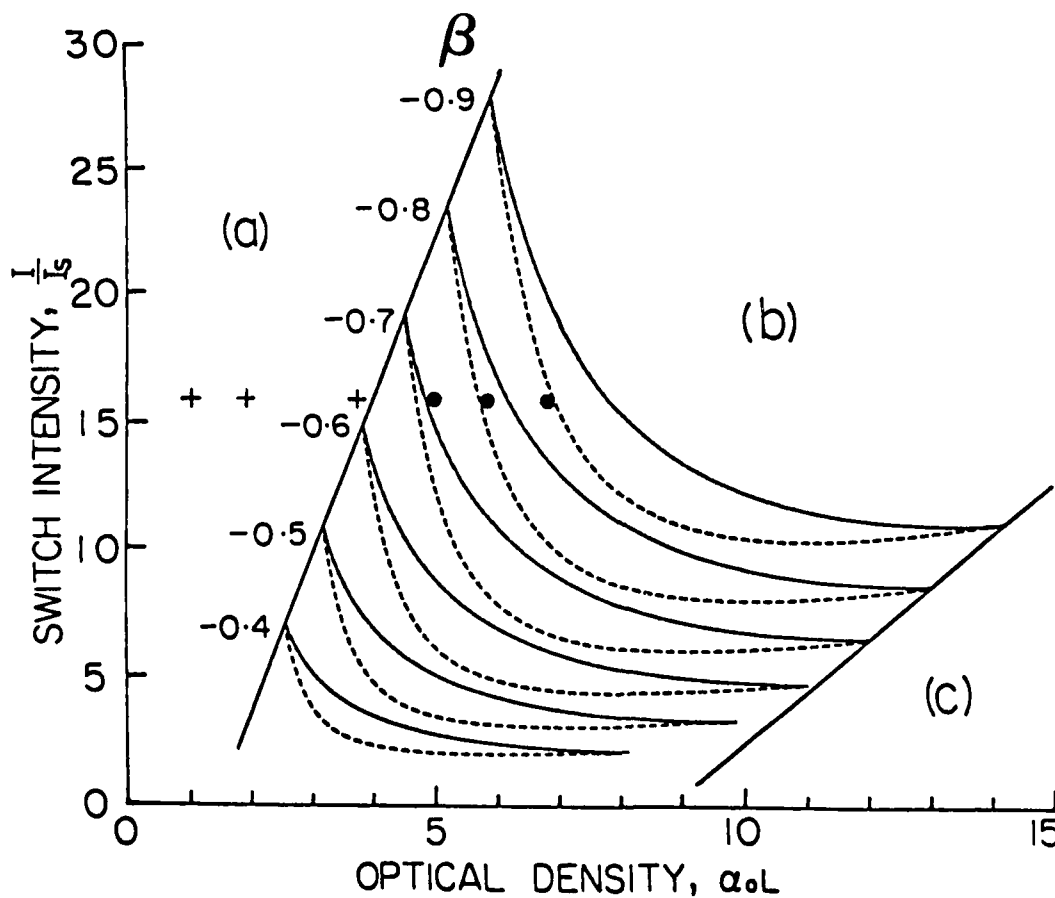


Figure 2. Normalized intensities for switch-on (solid curves) and switch-off (dashed curves) as a function of the initial optical density of BDN dye in a Fabry-Perot with detuning  $\beta$  as a parameter. Circles represent points at which bistability was observed experimentally and +'s represent points at which bistability was not observed.



## TRAVELLING WAVE BISTABILITY

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In the majority of optically bistable systems discussed to date, the bistability results from nonuniqueness in the boundary conditions coupled with a unique wave equation. We describe here optical bistability which occurs with unique boundary conditions but a non-unique wave equation, due to bistability in the susceptibility tensor.

Bistability in the susceptibility tensor may be seen classically by studying the Duffing oscillator, which displays a bistable amplitude response to an external periodic driving source.<sup>1</sup> This oscillator is an harmonic oscillator with a small cubic nonlinearity in the restoring force and suggests that atomic or molecular systems which contain quartic potential terms should display an intrinsic bistable susceptibility under appropriate conditions.

The essential features of the quantum mechanical calculation are demonstrated in the classical case. For simplicity we ignore damping terms here, but will include these terms in the general results which we will present. The Duffing oscillator under a sinusoidal driving force obeys the following equation:

$$d^2x/dt^2 = -\alpha x - \beta x^3 + F \cos \omega t. \quad (1)$$

Duffing's iteration procedure begins with a first approximation of  $x_0(t) = A \cos \omega t$ , which is placed in the right hand side of eq. (1), which is then solved for  $x_1(t)$ . This leads to

$$x_1(t) = A_1 \cos \omega t + (\beta A^3 / 36 \omega^2) \cos 3 \omega t \quad (2)$$

$$\text{where } A_1 = \omega^{-2} (\alpha A + (3/4) \beta A^3 - F), \quad (3)$$

Duffing argues that  $A_1$  should differ only slightly from  $A$  if  $\beta$  is small, and thus to good approximation eq. (3) becomes

$$\omega^2 = \alpha + (3/4) \beta A^2 - F/A. \quad (4)$$

This avoids the problem of standard iteration procedures which assume  $A$  is a function of  $\omega$  and which make it difficult, if not impossible, to obtain the most striking features of the  $A$  vs.  $\omega$  response curves. From eq. (4) one can show that over a range of  $\omega$  there are two stable and one unstable values of  $A$  for each  $\omega$ . Keeping  $\omega$  fixed and varying the amplitude of the driving force leads to a bistable amplitude response vs. driving force curve identical to those common in bistability.

Classically, the susceptibility of the medium is proportional to the (average) amplitude of the driven oscillators. Thus the Duffing oscillator classically may be expected to lead to optical travelling wave bistability.

We are in the process of performing a quantum mechanical calculation of the Duffing oscillator. We are using the density matrix approach and first calculate to order  $\beta$  corrections to the quantum harmonic oscillator due to a time independent perturbation hamiltonian of the form  $H' = \beta x^4$ . With the new wave functions and energy levels, we calculate the susceptibility  $\chi = \chi(1) + \chi(2) + \chi(3) + \dots$  to third order in a density matrix iteration.

Results of the classical calculation suggest that, at the very least, a travelling wave dispersive bistability should

appear which would be observed as a phase bistability at the fundamental frequency in a wave travelling through such a medium. This can be converted to an intensity bistability through interference with a reference beam, or as a polarization rotation in a nonlinear anisotropic crystal. In addition to these possibilities, our general expressions permit investigations of possible bistable absorption and purely quantum mechanical effects for systems close to their ground states. The effects of bistability on other nonlinear optical processes may also be studied via this method, and we shall report on our progress in these areas as well.

This work was supported by the National Science Foundation.

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# QUANTUM THEORY OF OPTICAL EXCITONIC BISTABILITY

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## Summary

A fully quantum mechanical formulation of the electron-hole-exciton-light system is developed following the work of Hanamura. In this formulation

1. The excitons are described by the "bosonisation transformation of Marumori *et al.* In this method exciton bose operators are introduced to describe electron-hole pairs, and antisymmetrisation effects are taken account of by a unitary transformation of the system operators, which introduces extra nonlinearities into the hamiltonian. This means that exclusion principle effects at high exciton densities are fully accounted for. The theory is formulated for Wannier (non-localised) excitons.
2. Damping of light and excitons is accounted for by inclusion of heat baths which are eliminated to give a Q.M. master equation for the photon-exciton system alone.
3. It is assumed that only one (or a few) exciton modes are coupled strongly to the light field, and other modes are eliminated as a heat bath. Optical Bistability occurs at both low densities and high densities, and arises from distinct mechanisms.
4. Low Density Bistability is a new prediction. The mechanism is that of nonlinearities in refractive index and absorption which arise from the exchange force between excitons.
5. High Density Bistability arises when exciton density is very large, and bose condensation effects are noticeable in the mode of interest. Switching occurs between an upper branch, described by the high density

theory, and a lower branch described by the low density theory. This corresponds to the experiments of Gibbs *et al.*

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OPTICAL BISTABILITY IN SEMICONDUCTORS

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Summary

There are two different models to explain the observed optical bistability (OB) in GaAs. Haken and Goll<sup>1</sup> use the saturation of the two-level (electron and hole) atoms as the mechanism of OB, whereas the Stark shift of excitons is the dominant factor in the work by Koch and Haug.<sup>2</sup> We analyze both models, including the local field effect.<sup>3</sup> It is shown that the validity of each model depends very much upon the parameters involved. We also give a similar analysis for CuCl where the biexcitons are important. In contrast to the earlier work by Abram and Maruani,<sup>4</sup> we find by careful analysis, including the origin of the background dielectric constant, that the local field correction is not important in this case. The contribution of the exciton and biexciton interaction, however, can be important and should be included in the calculation of OB given by Haug.<sup>5</sup> The changes in the numerical results of Ref. 5, by including the exciton-biexciton interaction and the local field correction are to be represented.

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## Critical Slowing-Down in Microwave Absorptive Bistability

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In this experiment one utilizes a microwave confocal open resonator in the transmission configuration, having a  $Q$  factor of  $5 \cdot 10^5$ . The resonator is filled with ammonia gas at the pressure of 1 mTorr. A klystron oscillator, frequency stabilized on the 3-3 ammonia inversion line, feeds the resonator which is tuned at the same frequency. In these experimental conditions, if the input power  $P_I$  is continuously increased from a low to a sufficiently high value, the output power changes abruptly from a low to a high value. For a fixed gas pressure, this change occurs at a fixed threshold input power  $P_{up}$ . The same occurs if the input power is continuously decreased from a high value; in this case one has  $P_{down} < P_{up}$ ,  $P_{down}$  being the threshold power at which the system switches from the high to the low transmission state.

If the input power is suddenly changed from a low value to a value  $P_I = P_{up} + \Delta P$  with  $\Delta P / P_{up} \ll 1$ , the transmitted power  $P_T$  does not follow immediately this change, but there is a time delay  $\tau$ , which may be of the order of several milliseconds in the experimental conditions reported, with  $\lim (\Delta P \rightarrow 0) = \infty$ .

This effect, called "critical slowing-down", is observed also in the downward transition, when the input power is suddenly changed from a high value to  $P_I = P_{\text{down}} - \Delta P$ .

A theoretical approach to this phenomenon may be given, in which the behaviour of the system is described writing two differential equations. The first equation gives the time evolution of the energy stored into the resonator filled with the absorbing gas. The second describes the time evolution of the difference of population of the two states, between which the microwave transition occurs. These two equations are coupled because the gas absorption coefficient appearing in the first one depends on the population difference, which depends on the stored energy. The numerical solution of this system of differential equations, gives account of the experimental observations with good agreement. In these calculations the gaussian distribution of the electromagnetic field inside the resonator has been considered, but the influence of the absorbing gas with respect to this distribution has been neglected.

Absorptive Optical Bistability with Laser  
Amplitude Fluctuations

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In this paper we discuss the influence of laser fluctuations on the hysteresis cycle in Optical Bistability. For the simplest case of an Absorptive Optical Bistability in a good cavity driven by an external laser light with non-white Gaussian amplitude fluctuations we derive a Fokker-Planck equation in the limit of large laser linewidth and good amplitude stabilization. This Fokker-Planck equation has a nonconstant diffusion term which shifts the most probable values of the steady state probability distribution from its deterministic positions. From the form of the steady state solution of the Fokker-Planck equation with non-white amplitude fluctuations we discuss the relative stability of the bistable branches. As in the case of quantum fluctuations the random amplitude of the laser field leads to a small range of values of the injected laser intensity in which the two bistable peaks have a comparable area. The transmitted field averaged with respect to the stationary distribution function coincides with one of the two deterministic branches except on the narrow transition region where we have large fluctuations.

PRECISE MEASUREMENTS OF BISTABILITY IN A  $\text{CO}_2$  LASER WITH  
INTRACAVITY SATURABLE ABSORBER

by

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Optical bistability in a laser with an intracavity saturable absorber is a phenomenon theoretically investigated firstly by Lugiato et al<sup>(1)</sup>. The experimental observations in  $\text{CO}_2$  lasers<sup>(2,3)</sup> have provided clear evidence of the bistability phenomenon and shown that the absorber parameters could be determined from these observations. In this work a quantitative comparison between a theoretical description of optical bistability and the experimental results is presented for the first time. The parameters defining the laser cavity and the  $\text{CO}_2$  amplifying medium were measured carefully, and the bistability was observed and analyzed quantitatively as a function of the laser parameters.

The dashed line of Figure I reports the measured output power ( in mW ) of the  $\text{CO}_2$  laser ( operating on the 10P(10) line ) versus the amplifier current (in mA ) in absence of the intracavity absorber. The continuous line represents the output power when 25 mTorr pressure  $\text{SF}_6$  gas was introduced in the absorber cell, with a bistable behaviour extending over a large range of the monitored region.

The laser parameters were determined looking at the

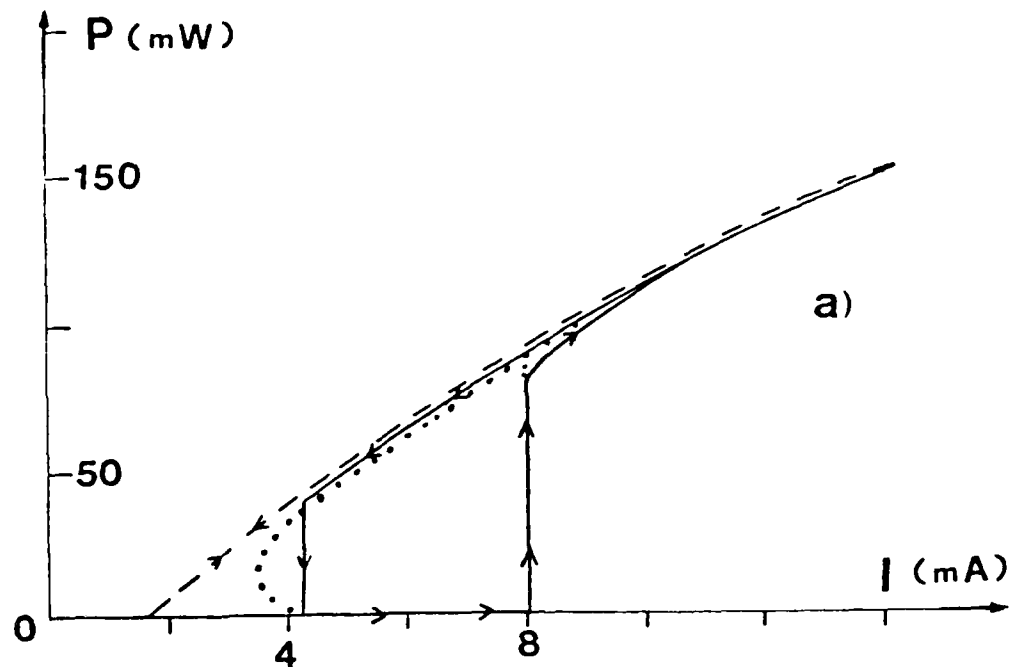


Figure I

output power with the absorber cell filled with ammonia diluted in helium, that behaves as an unsaturable absorber in the range of available intracavity power. Making use of the Rigrod equation<sup>(4)</sup> to describe this laser operation we have derived the true reflectances of the laser end mirrors  $r_1 = 0.95(3)$ ,  $r_2 = 0.82(3)$  and the amplifier saturation parameter  $I_a = 50(5) \text{ W/cm}^2$ .

The bistability observations were quantitatively analyzed by extending the Rigrod model to describe the laser operation with an intracavity saturable absorber. Thus we have properly taken into account the mirror losses and the exponential growth and attenuation in the amplifier and absorber respectively. On the contrary a mean field approximation has been introduced to describe the standing wave operation: the gain and absorption coefficients have been calculated through an average over the wavelength of the laser radiation. The steady state laser output power has been determined graphically from the operating point.

of two curves describing the laser operation. The dotted line of the Figure represents the results obtained from our steady state equation when the relative saturability of the absorber and amplifier was fixed to 60. It is important to notice that although an homogeneous broadening was introduced to describe the low-pressure inhomogeneously broadened absorber, the agreement between the theoretical fit and the experimental results is very good.

Our model as well the theory of ref (1), valid for small gain and losses, do not include the frequency pulling effects produced by the mismatch of the amplifier and absorber frequencies. We have experimentally verified that the laser frequency plays a negligible role on the laser behaviour reported in the Figure. By beating the laser output against a reference laser we have observed that in absence of the absorber a laser frequency shift smaller than 5 MHz occurs near threshold in the 2-3 mA range, but no shift occurs at larger currents. When the absorber is introduced no laser frequency shift occurs over the bistability region.

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**Periodic window, period doubling, and chaos in a liquid crystal bistable optical device**

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A twisted nematic liquid crystal (TNLC) bistable optical device is constructed to study the Feigenbaum scenario<sup>1</sup> in a manner of Gibbs et al.<sup>2</sup> The simple transmission, having a single near-trapezoidal hump and a non-negligible extinction level, facilitates experimental observation and computer simulation. A number of periodic windows, i.e. 3, 4, 5, and 7T (T means a period), have been easily observed as well as period doublings up to 4T and chaos. It was also confirmed that these waveforms have the 'universal-sequence' patterns reported by Metropolis et al.<sup>3</sup> Based on a simple 'broken-linear' modelling of the TNLC modulator transmission characteristics, the computer simulation has been performed to show period doublings, periodic windows, and period mergings. It exhibits reasonably good agreement with the experiment, and explains

easy observation of periodic windows.

Fig.1 shows the schematic of the experimental set-up, which is similar to that of Ref.2. The laser input intensity is controlled by a variable attenuator. The detector output voltage is delayed at the delay buffer by the time  $t_R$ . The adder output signal  $\tilde{V}$  is converted by the dc to ac convertor, and fed to the modulator. The dc to ac convertor produces the 10 kHz square wave output, whose rms value is equal to the absolute value of its input  $\tilde{V}$ . Thus our TNLC modulator responds to the absolute value of  $\tilde{V}$ . Fig.2 shows the measured transmission characteristics  $T(\tilde{V})$  of the TNLC modulator with respect to the absolute value of  $\tilde{V}$ . Allowing  $\tilde{V}$  to take also the negative value by a proper negative bias  $V_b$ , we can obtain a single near-trapezoidal hump in the transmission characteristics.

The equation governing this device may be simply modeled as<sup>2</sup>

$$\tau dV(t)/dt + V(t) = aT(\tilde{V}), \quad \tilde{V} = g(t - t_R) + V_b, \quad (1)$$

where  $V(t)$  is the detector output voltage,  $\tau$  the response time of the device,  $g$  the total gain of two dc amplifiers, and  $a$  is the bifurcation parameter. Dropping the term  $\tau dV(t)/dt$  in Eq.(1) under the condition  $\tau \ll t_R$ , and defining  $t_n = nt_R$  and  $V_n = V(t_n)$ , we obtain<sup>2</sup>

$$V_{n+1} = aT(gV_n + V_b). \quad (2)$$

For simplicity in computer simulation  $T(\tilde{V})$  is approximated by the dashed lines as shown in Fig.2, and the bias voltage  $V_b$  is chosen as  $-1.43V$ . The computer simulation utilizing these data shows period doublings, periodic windows, and period mergings. It also shows the wide periodic window  $3T$  due to the non-negligible  $T_{min}$ .

Some oscillograms of the detector output waveforms observed in the experiment are shown in Fig.4. Three pictures represent (a)  $2T$ , (b)  $4T$ , and (c)  $3T$ , respectively. The first two cases i.e.  $2T$  and  $4T$  result from the bifurcations of the stable state,  $1T$ . The last one is the periodic window,  $3T$ . The measured value of this window size is very wide, which is predicted by the computer simulation.

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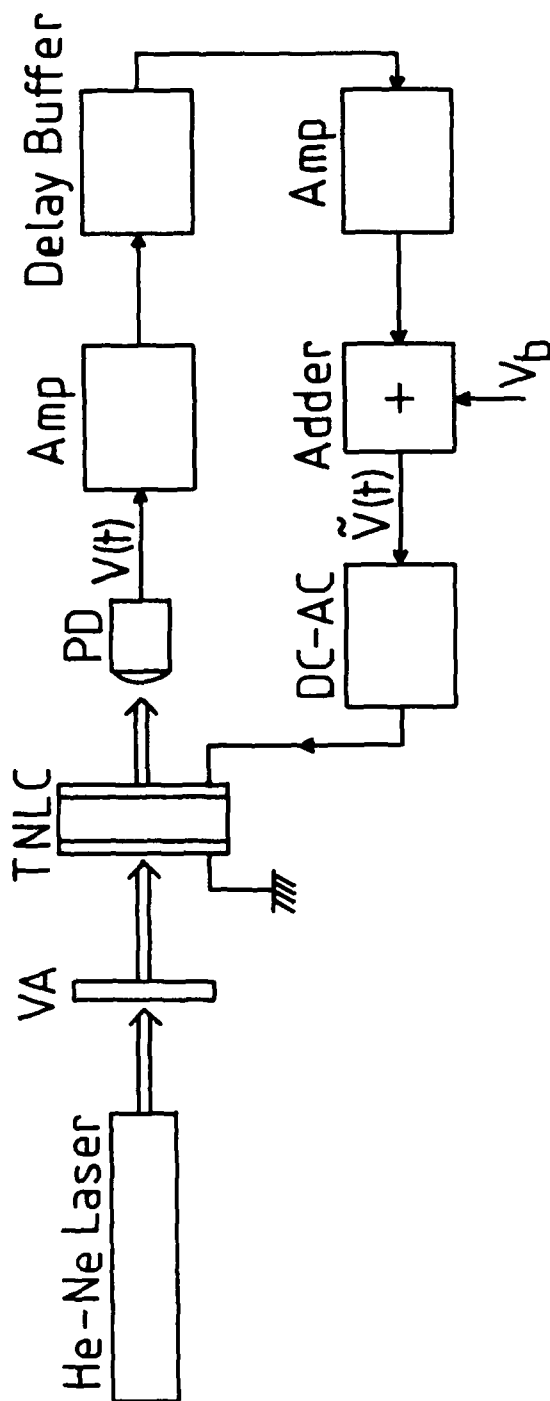


Fig.1 Experimental set-up

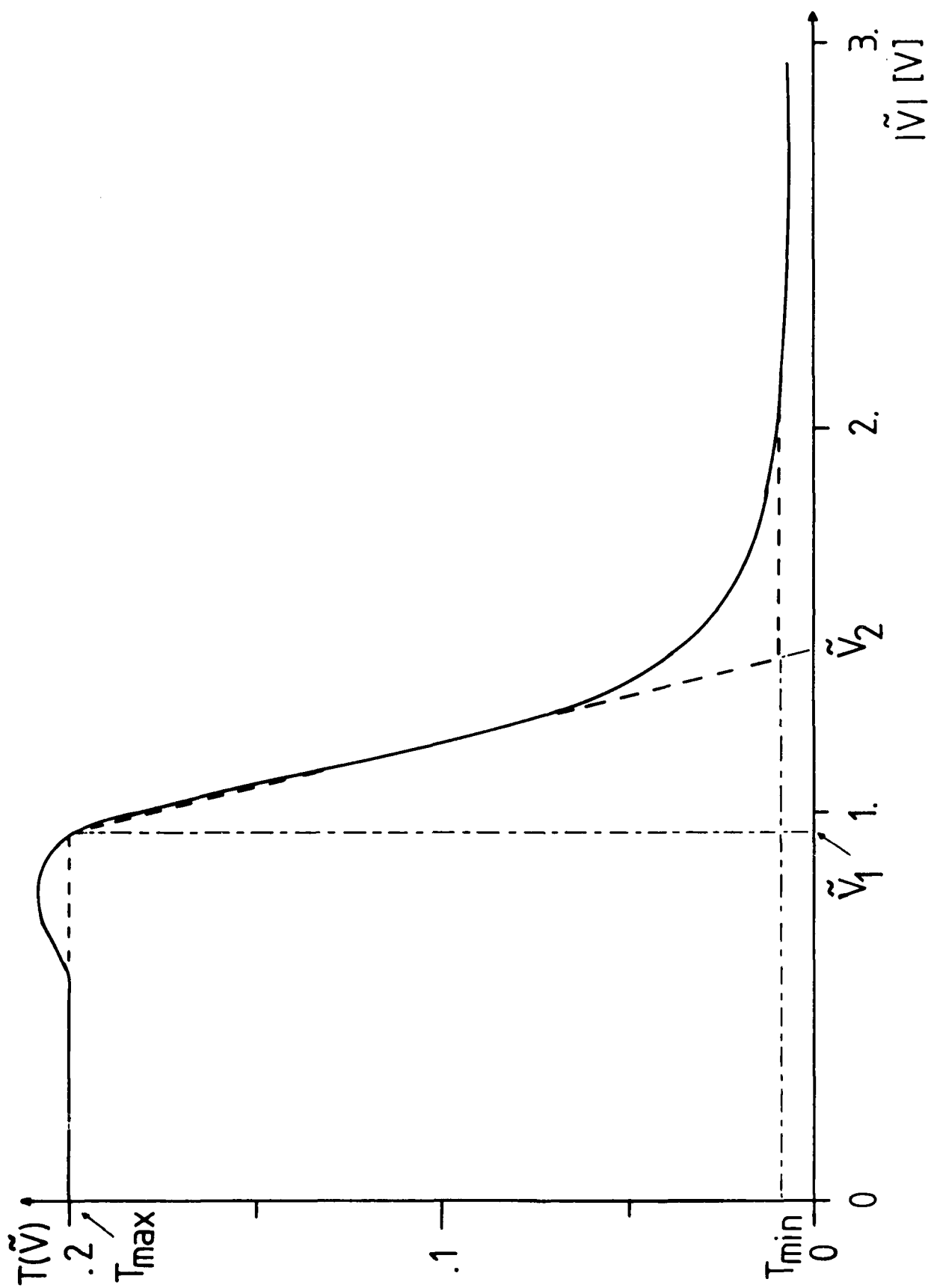
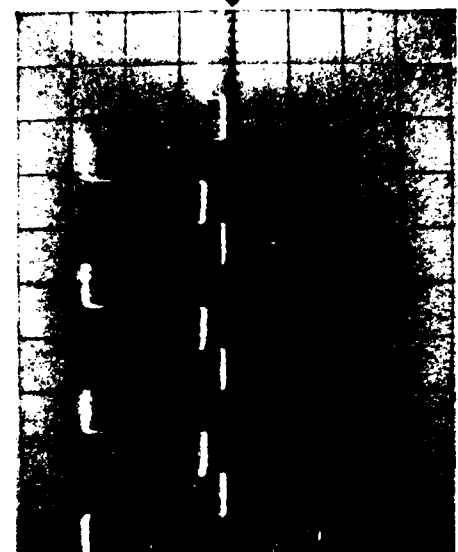
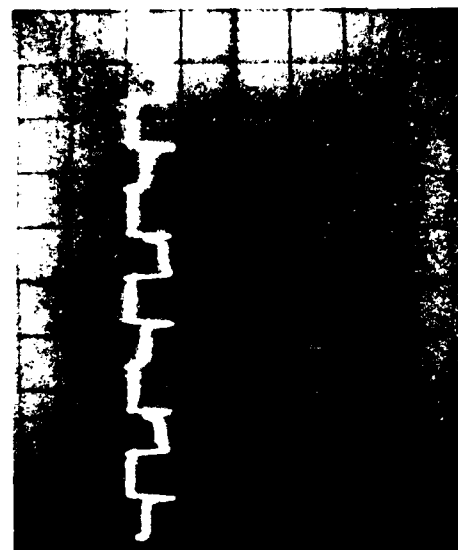
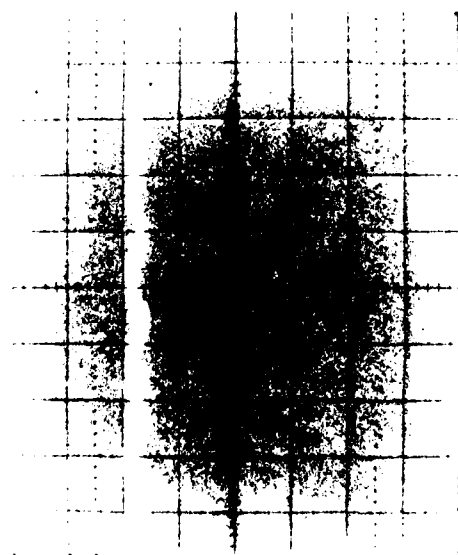


Fig.2 Transmission of the TNLC modulator



ThB33-6

Fig. 3 Detector output oscillograms. vert: 50mV/div, hori: 5 sec/div.

Effects of the Radial Variation of the Electric Field on  
Some Instabilities in Lasers and in Optical Bistability.

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In 1966 Haken and Risken, Schmid and Weidlich<sup>(1)</sup> showed that under proper conditions the stationary state of a homogeneously broadened, single mode ring laser can become unstable. The instability requires a "bad cavity" situation, i.e. the cavity damping constant must be larger than the atomic decay rates. Later, Haken<sup>(2)</sup> proved that such an instability coincides with the well known Lorenz instability, which leads to chaotic behaviour. On the other hand, Risken and Nummedal and independently Graham and Haken<sup>(3)</sup> analyzed the homogeneously broadened ring laser taking into account all the longitudinal cavity modes. They showed that under appropriate conditions in the good cavity case some cavity modes, different from the one resonant with the atomic system, can become unstable. In this case, one has the formation of a pulse that travels in the cavity. In 1978, Bonifacio and Lugiato<sup>(4)</sup> showed that a similar self-pulsing behaviour in good cavity conditions occurs in optical bistability in a ring cavity filled with a homogeneously broadened atomic sample. In all of Refs 1-4 the electric field is treated in the plane wave

approximation. However, in a real laser the electric field has a radial profile which is typically gaussian, hence one naturally asks what is the effect of this transverse structure on these instabilities. In this paper, we assume that the electric field has the radial profile of the  $TEM_{00}$  mode of a ring cavity with spherical mirrors. In the case of optical bistability, we assume that the incident field also corresponds to the  $TEM_{00}$  mode. We introduce the hypotheses that i) the absorption parameter  $\alpha L$ , with  $L$  = length of the atomic sample, and the mirror transmissivity coefficient  $T$  are both small, with  $C = \alpha L/2T$  arbitrary, ii) the Fresnel number  $F = \pi w_0^2 / \lambda_0 L$ , where  $w_0$  is the beam waist and  $\lambda_0$  the wavelength, is large, iii) the atomic system is two-level and homogeneously broadened. In the case that the transverse dimension  $d$  of the atomic sample is much smaller than the beam waist  $w_0$  so that only the central part of the beam interacts with the atoms, we are practically in plane wave conditions and therefore we recover the instability condition of the plane wave theory. In the opposite case  $d \gg w_0$ , we find that all the instabilities found in Refs. 1-4 both in the good and in the bad cavity case, are washed out by the integral over the radial variable. While in the laser case this result seems to exclude that such instabilities can be experimentally observed with homogeneous broadening in the usual situation  $d \gg w_0$ , in the case of optical bistability the hypothesis that the field internal to the filled cavity corresponds to a  $TEM_{00}$  mode is questionable at least when  $F \gg 1$ .<sup>(5)</sup>



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THEORY OF OPTICAL BISTABILITY IN  
COLLINEAR DEGENERATE FOUR-WAVE MIXING

By

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SUMMARY

Optical bistability occurs in many all-optical devices with non-linear couplings and feedback mechanisms. Thus, bistability is predicted for phase conjugate resonators,<sup>1</sup> Fabry-Perot interferometers,<sup>2</sup> and, generally wave mixing devices with a source of feedback. Bistability is therefore expected in four-wave mixing, where the Maxwell equations provide nonlinear feedback through the nonlinear susceptibility.

The purpose of this paper is to present and solve a complete theory of optical bistability in collinear degenerate four-wave mixing in isotropic media. Optical bistability has been previously predicted in collinear degenerate four-wave mixing in isotropic media.<sup>3</sup> This paper extends the incomplete theory of reference 3 to include all phase-matched contributions to the equations of motion for the slow-varying amplitudes. The additional non-conjugate contributions<sup>4</sup> significantly alter the physics of the interactions and provide additional feedback among the various pump and signal waves. New predictions for bistability in the reflection coefficient are made and illustrated for some practical cases.

The full analysis is repeated for collinear degenerate four-wave mixing in absorbing media.<sup>5</sup> Here, the slowly-varying envelope approximation (SVEA) is achieved by equating the coefficients of all Fourier coefficients appearing in the Maxwell equations, and inverting a four by four matrix to get exact SVEA equations. This technique avoids ambiguities<sup>5</sup> in the SVEA and provides a correct treatment of four-wave mixing in absorbing media. Optical bistability is predicted for these media, and specific examples are presented.

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A Saturable Interferometer Theory  
of Absorptive Optical Bistability

by

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In this paper we will utilize the method of summing transmitted waves routinely used to analyze interferometer structures to develop an OB equation. The method yields a different state equation than the usual mean-field model. However, for small  $C$  values and/or large input intensities a Taylor expansion of our state equation yields the mean field pure absorptive OB equation. Our equation also gives a different condition for hysteresis. Due to the simplicity of this approach which utilizes the sum of a geometric series, we are able to interrogate the system after any given number of round trips via the analytic expression for the sum at that point. This allows us to obtain an inverted function of time and to determine the time evolution of the system. Using these expressions, we are able to give switching times for both the up and down processes.

The theory we present is based on a self consistent sum of partial waves emitted by a resonant structure. In these derivations we consider only the zero detuning case and therefore assume that there are only hysteresis effects due to  $\chi_2$  saturation. We will solve the case of a Fabry-Perot resonator with plane mirrors having power transmission coefficient  $T$ . The round trip length of the cavity is taken to be  $2L$  and the cavity is filled by a medium having an unsaturated absorption coefficient  $\alpha_0$ .

The medium is assumed to be an ideal homogeneously broadened two level system with a saturation intensity  $I_s$  given by:

$$I_s = \frac{\gamma_{11} \gamma_{22} \hbar^2 c}{2\mu^2} \quad (1)$$

In terms of  $I_s$ , the absorption coefficient is given by

$$\alpha = \frac{\alpha_0}{1 + I/I_s} \quad (2)$$

where  $I$  is the instantaneous spatially averaged intensity inside the Fabry-Perot.

Using the usual sum of partial waves method utilized for analyzing interferometers, we can express the transmitted electric field:

$$X = TYe^{ik\ell} \exp \left\{ \frac{\rho_0}{1+X^2} \right\} \sum_{m=0}^M (1-T)^m \exp \left\{ 2m \left( ik - \frac{\rho_0}{1+X^2} \right) \right\} \quad (3)$$

$\rho_0 = \alpha_c \ell$  and  $Y$  and  $X$  are the usual dimensionless incident and transmitted electric fields. This expression can be made self consistent by realizing that  $I$  or  $E^2$  may be related to  $E_T$  in the limit of spatial averaging. In the steady-state limit when  $M \rightarrow \infty$ , the sum may be evaluated using the geometric series limit and the self consistent equation which results is given by:

$$TY = X[2\sinh(\rho) + Te^{-\rho}] \quad (4)$$

where  $\rho = \frac{\rho_0}{1+X^2}$ . The transmitted intensity as a function of the incident intensity is shown in figure 1. Figure 2 shows a comparison of our result with that of the mean field model for small  $\rho_0(1,2)$ . A Taylor expansion of the exponentials in equation (3) results in the usual mean field model cubic equation. The condition for obtaining bistability for this equation in terms of  $\rho_0$  and  $T$  is graphically shown in figure 3. From the results it is evident that this formulation admits a larger range of bistability conditions.

The time evolution in OB has been studied numerically and from the standpoint of asymptotic growth rates resulting from stability analysis. (3,4) Mandel and Erneux have shown interesting results concerning the switching times in the limit of  $C$  large. (5,6)

An expression which relates the output intensity  $X$  after a given number of round trips  $M$  to the input intensity  $Y$  can be found from equation (3). This is accomplished via the expression for the value of a finite geometric sum. This results in:

$$MT_c = \frac{\ln \left[ \frac{1}{R} \left( e^{2\rho} - \frac{(e^{3\rho} - Re^{\rho})}{T} \cdot \frac{X}{Y} \right) \right]}{\ln(1-T) - 2\rho} \quad (5)$$

Equation (5) gives an expression for the time in cavity round trips it takes to develop a transmitted field  $X$  when a field  $Y$  is instantaneously present at the first mirror of the structure. The results for two values of the input intensity which exceed the switch up value are shown in figure 4. The upper curve ( $Y^2 = 60$ ) indicates that approximately 25-30 round trips are required and that the time evolution is a direct trajectory towards the upper branch value ( $38 = X^2$  in this case). The lower solid curve shows the results of equation (5) when the input value just barely exceeds the up switch value of  $Y^2 = 30$ . The point labelled A represents the largest deviation from the dashed curve. This point corresponds to the value of  $X^2$  on the hysteresis loop which comes closest to the line  $Y^2 = 35$  in the  $X^2$ - $Y^2$  plane. Thus it appears that the unstable points can act as attractors for solutions which lie on only the upper branch if the input intensity is near the switch up value. The time evolving system is drawn in towards the lower branch as it approaches its steady state value in the upper branch. Equation (5) can be used

to study the effect of the controllable physical parameters  $\rho_0$  and  $T$  on the up switch and down switching times for the optically bistable system. This is to our knowledge the only analytic expression which demonstrates this attractor behavior explicitly in the time domain.

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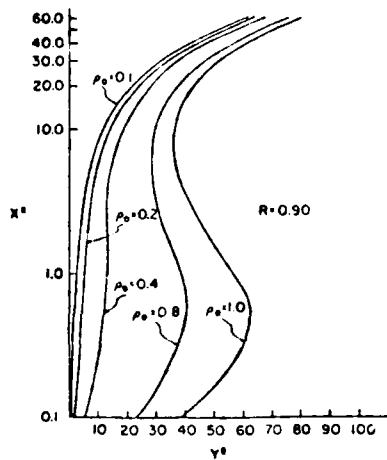


Figure 1. Hysteresis curves for a Fabry-Perot attractor.

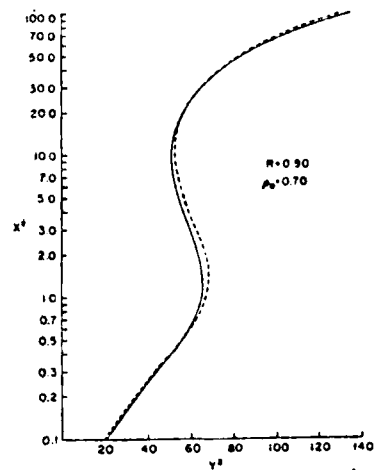


Figure 2. Comparison of the self consistent interferometer results with those of the mean field model. The mean field results are represented by the dashed curve.

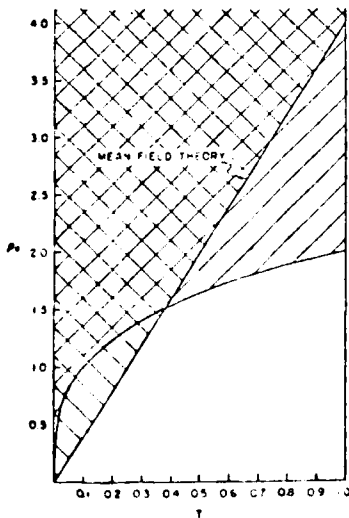


Figure 3. Comparison of the regimes of bistability for the two results.

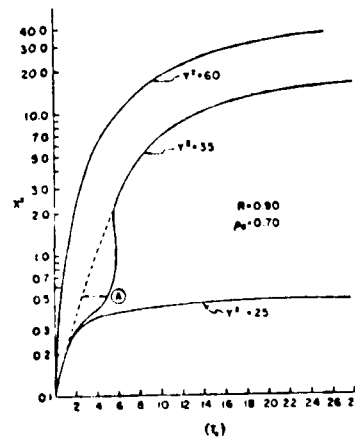


Figure 4. Dynamic up-switching behavior for two input intensities exceeding the switch-up value. The  $y^2 = 35$  curve shows the attractor behavior of the unstable switching point in the time evolution of the system.

## BISTABILITY IN OPTICAL WAVEGUIDES

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Using a treatment due to Bendow, Gianino and Tzoar (1) in which the electric field of a cw gaussian beam in a nonlinear waveguide is given utilizing the paraxial approximation, it is shown that a bistable behavior can occur in a cylindrically symmetric nonlinear waveguide in which the dielectric constant has a parabolic behavior in the radial direction.

The treatment uses Bendow et al.'s method of expansion of the electric field in a set of "quasi-nonlinear" modes of the guide obtained within the spirit of paraxial approximation.

It is shown that at each chosen length of the waveguide it is possible, in general, to find a set of waveguide parameters which give rise to a bistable behavior in the light intensity coming out at that section.

The conditions that the waveguide parameters must fulfill to give a bistable behavior are discussed. The effect is strongly dependent on the transverse inhomogeneity of the waveguide, and is absent for homogeneous structures.

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## NON-LINEAR FABRY-PEROT TRANSMISSION IN A CdHgTe ETALON

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## SUMMARY

We have recently reported the observation of large intensity dependent changes in the refractive index of cadmium mercury telluride at 175K and  $10.6 \mu\text{m}$  [1]. This non-linearity arises because of bandgap saturation, as seen in InSb [2], and through a free carrier, plasma contribution. In contrast to the InSb non-linearity which was modelled in terms of an " $n_2 I$ " correction to the linear refractive index ( $I$  is the incident intensity), the non-linearity in the narrower bandgap CdHgTe at 175K showed an intensity dependence proportional to  $I^{1/3}$ . This was consistent with the dominant Auger recombination process.

In this paper we consider the implication of this result for optical bistability or non-linear transmission using CdHgTe. Although the magnitude of the non-linearity at low incident intensities is greater than that observed in InSb, the  $I^{1/3}$  dependence causes the refractive index change to become relatively much weaker at intensities  $> 1 \text{ W/cm}^2$ . This has a marked effect on the transmission characteristics of an etalon - the separation between Fabry-Perot orders becomes increasingly large and the first order behaviour is strongly dependent on the initial detuning. The  $I^{1/3}$  dependence arises from the nature of the recombination process and may occur in other semiconductors over particular temperature ranges (eg InSb at room temperature). We will discuss the implications of our work for optimum choice of materials for bistability. The paper will include both experimental and theoretical results.

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Summary:

Homogeneously broadened single mode lasers are well known to exhibit self-pulsing instabilities and chaotic dynamics under the combined conditions of large ratio of gain over losses and low cavity quality. In fact, a homogeneously broadened single mode laser in resonance has essentially 3 degrees of freedom only and is realistically described by the Lorenz model, which has served as a prototype model for investigating chaos in continuous dynamical systems. However, the above mentioned conditions for the occurrence of chaos have not yet been realized, experimentally. Recently, bad-cavity instabilities have also been discussed for inhomogeneously broadened lasers, and seem to be more easily accessible, experimentally, but models of comparable simplicity as the Lorenz model have not yet been proposed for inhomogeneously broadened lasers. It is the purpose of this contribution to show how such models can be constructed. We find that an infinite hierarchy of models exist which increase in accuracy and complexity. We present a 4-dimensional model and a 6-dimensional model, which are the two simplest members of this hierarchy. The results of a linear stability of the time-independent states and some numerical solutions are given to show the various types of dynamical behavior which may occur in these models. The dynamical behavior is found to be much richer than in the homogeneously broadened case and is obtained under phy-

sically more realistic conditions.

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SELF-PULSINGS, HYSTERESIS AND CHAOS IN  
A STANDING WAVE LASER

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Based on the system of semi-classical equations analysis and the numerical experiment, the temporal nature of the standing wave laser generation has been studied. It is shown that such a laser is a dynamic system in which, along with the selforganization processes leading to regular continuous pulsings /1/, stochastization of the strange attractor type is possible /2/. This diversified behaviour is due to the competition and interrelation of factors accounting for the steady-state behaviour instability. These factors are the development of nutation oscillations due to the coherent properties of the active medium and interaction of counter-running waves on the gain and losses inhomogeneities in the cavity differing in physical nature.

In particular, the steady-state behaviour instability is responsible for the appearance of continuous pulsings with the period of  $2L/CS$  ( $L$  is the cavity length,  $S = 1, 2, \dots$ ). The change of parameters, for example, an increase in the pump rate causes a succession of bifurcations to take place, which result in the appearance of pulsings with a new value of  $S$ . The order of changing  $S$  depends on the position of the active medium in the cavity, the mirrors' reflectivity and other laser parameters. Under certain conditions, excitation of self-pulsings with different periods is possible in the system. In this

case hysteresis can be observed /3/, whose essence is in that, depending on the prehistory, a sequence of stable ultrashort radiation pulses with different periods can be generated in the laser. For example, with the same parameters the system generates either one or two pulses during the round trip of radiation in the cavity. The pump instability in time required for such hysteresis can be achieved experimentally.

When the self-oscillations with different periods are unstable random motion will take place, whose nature is determined by the main cause of the steady-state instability. In particular, it has been found that, according to the mechanism of the ultrashort pulses formation, their regularity is affected either when the field phase is constant or when it undergoes a chaotic change indicative of a chaotic change of oscillations at different steady states. When the laser parameters correspond to the laser spiking low frequency pulsing stochastization is possible, the fine temporal structure remaining regular. The role of coherent and incoherent processes and the spatial population grating in the formation of such stochastic behaviour in the laser as a dynamic system with determined parameters has been defined.

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THE NMR LASER - A NONLINEAR SOLID STATE SYSTEM SHOWING CHAOS

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Pathways from simple to complex behavior of nonlinear systems is a topic of intense theoretical and experimental interest after J.M. Feigenbaum, J.P. Eckmann and others have demonstrated universal features of a sequence of period-doubling bifurcations leading to chaotic states.

In this communication we shall report on novel experimental observations and corresponding theoretical considerations of a system of strongly polarized nuclear spins in a solid which is placed inside the tuned coil of an NMR detector such that self-induced laser activity sets in. I will present experimental evidence for sequences of subharmonic bifurcations, transitions to chaotic behavior, the finely interlaced structure of separated basins of attraction, hysteresis, intermittency, and a new type of bistability for driven laser systems. Then I shall confront the observed facts with solutions of numerically tractable Bloch type order parameter differential equations which indicate that various normal and inverted Feigenbaum cascades of bifurcations may exist in a driven laser if appropriate values of the physical system parameters are selected and controlled to a high degree of accuracy.

**Optical Nonlinearity and Bistability due to the Biexciton  
Two-Photon Resonance in CuCl**

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It is well established that biexcitons can be generated directly by two-photon absorption in CuCl. The intensity dependent nonlinearity arising from this resonance can be used to obtain optical bistability. Recently, Koch and Haug<sup>(1)</sup> and also Hanamura<sup>(2)</sup> have predicted the possibility of a bistable operation in a CuCl Fabry-Perot etalon. Hanamura uses a density matrix formalism and predicts that the switching can respond in a picosecond time scale due to the virtual formation of biexcitons. Koch and Haug obtain the complex dielectric function from a Green's function formalism and suggest a bistable behavior for a thin CuCl slab. They use a constant value for the biexciton linewidth,  $\gamma_{xx}$ , and a zero value for the exciton linewidth,  $\gamma_x$ . Their results indicate that a 1- $\mu\text{m}$ -thick CuCl etalon, with natural reflecting surfaces, can give rise to bistability with a switch-on intensity of about  $0.1 \text{ MW/cm}^2$ . The biexciton resonance, however, is located on the tail of the exciton resonance, which acts as a background absorption. Since this background absorption can be fatal to biexcitonic bistability, it should be taken into account by using the proper exciton linewidth. Also, the biexciton absorption line shows a remarkable broadening with input

laser intensity. This broadening should also be taken into account because it can modify the conditions required for achieving a bistable operation. The origin of the broadening has been the source of some controversy. Chase et al.<sup>(3)</sup> and also Itoh et al.<sup>(4)</sup> have attributed this effect to collisions between excitonic particles. This interpretation is supported by a more recent experiment performed by Peyghambarian et al.<sup>(5)</sup> in which the two-photon absorption resonance measured with two weak probe beams was broadened by simultaneous injection of biexcitons using an intense pump beam.

We use the complex dielectric function in the vicinity of the biexciton resonance as given in Ref. 1, together with the model developed for the collision broadening of the biexcitons.<sup>(3)</sup> This model takes into account liquid-like collisions in which the scattering rate is proportional to the ratio of the mean thermal velocity to the interparticle separation. It yields

$$\gamma_{xx} = \gamma_{xx,0} + \{[(E-E_{xx}/2)^4 + 4(K\sqrt{n_p})^4]^{1/2} - (E-E_{xx}/2)^2\}^{1/2} 2^{-1/2},$$

where  $K$  is a constant,  $\gamma_{xx,0}$  is the natural biexciton linewidth at low intensities,  $E_{xx}$  is the biexciton energy, and  $n_p$  is the polariton density.

By choosing  $\gamma_x = 0.03$  meV we find that the linear absorption at the tail of the exciton, near the biexciton resonance, is in good agreement with experimental values, as is the calculated value of the group velocity. The light intensity inside the etalon is obtained from

$$I = n_p E v_E,$$

where  $v_E$  is the energy velocity which is found to be close to the group velocity away from

the biexciton resonance.

The calculated absorption coefficient  $\alpha = 2Kn''$  and the change of refractive index  $\Delta n' = n'(1) - n'(0)$  are shown in Figs. 1 and 2, respectively, which are in agreement with recently published experimental results.<sup>4</sup> Figure 3 shows the bistability curves for a 10- $\mu\text{m}$ -thick CuCl slab having a reflectivity  $R = 0.9$  and  $R = 0.8$  at both surfaces where  $E = 3.177$  eV. One observes that bistability is lost at reflectivities smaller than 0.9 for this case. For a 1- $\mu\text{m}$ -thick sample bistability is theoretically possible only with extremely high intensities which will destroy the sample, and therefore are impractical to use. When the biexciton broadening is not taken into account, our results indicate that  $35 \text{ MW/cm}^2$  is needed to switch on a 1- $\mu\text{m}$  device with  $R = 0.9$  and about  $10 \text{ MW/cm}^2$  is needed for a 10- $\mu\text{m}$  device.

In summary, we show that unlike the previous prediction that a 1- $\mu\text{m}$  CuCl slab can be switched on with  $0.1 \text{ MW/cm}^2$  with uncoated surfaces, bistability can be obtained only with a high reflection coating of the Fabry-Perot etalon for  $\sim 10\text{-}\mu\text{m}$  samples, and with intensities on the order of  $10 \text{ MW/cm}^2$ .

We gratefully acknowledge support from NSF, AFOSR, and ARO.

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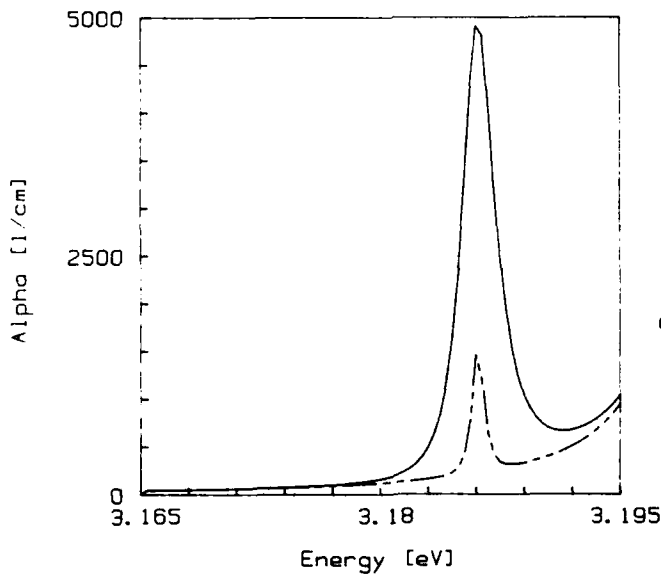


Figure 1.  $\alpha$  [ $\text{cm}^{-1}$ ] as a function of polariton energy  $E$  [eV] for  $n_p = 5 \times 10^{16} \text{ cm}^{-3}$  (full line) and  $n_p = 5 \times 10^{15} \text{ cm}^{-3}$  (dashed line), which corresponds to  $\sim 10 \text{ MW/cm}^2$  and  $\sim 1 \text{ MW/cm}^2$  at the biexciton resonance.

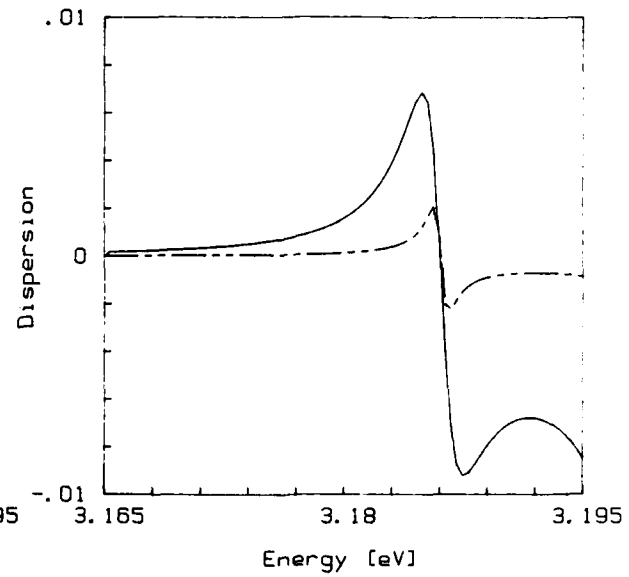


Figure 2.  $\Delta n'$  as a function of polariton energy  $E$  [eV] for the same parameters as in Fig. 1.

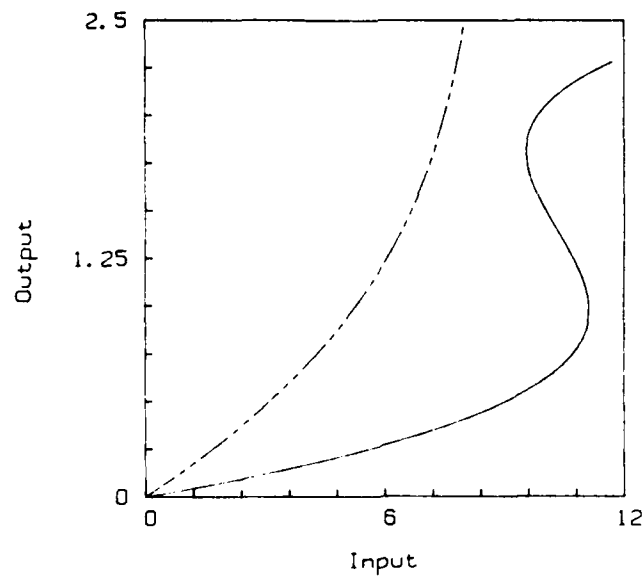


Figure 3. Input and output intensity [ $\text{MW/cm}^2$ ] of a 10- $\mu\text{m}$ -thick CuCl Fabry-Perot etalon at  $E = 3.177 \text{ eV}$  for  $R = 0.9$  (full line) and  $R = 0.8$  (dashed line.)

Optical extinction theorem theory of optical multi-stability  
bifurcations and turbulence in the Fabry-Perot interferometer

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We have reported two different aspects of this work already [1,2]. Nevertheless as the point of view is unconventional we review it here strictly within the context of the theory of optical bistability and optical multistability. We are concerned to connect the envelope Maxwell-Bloch equations with optical bistability (multistability) in a Fabry-Perot (FP) cavity in a rigorous and potentially quantitative way. One problem in this connection is an adequate statement about standing waves. We present methods which derive the standing wave equations of motion completely as a part of a comprehensive non-linear refractive index theory of multistability inside the FP cavity. The theory is a c-number one — but a comparable quantum theory seems possible. A key feature of the argument is generalisation of the famous 'optical extinction theorem' [1,2,3] to this non-linear regime. In practice it means we do not invoke any boundary conditions at the surfaces of the FP cavity — only conditions at infinity — and this offers advantages for the quantitative description as we show.

The Maxwell-Bloch equations are taken in fundamental (not envelope) form: the Maxwell equation is in integral form. In r.w.a. the Bloch equations are

$$\dot{r}_+(x,t) + \gamma_0(1+i\delta)r_+(x,t) = 2i\{E^*(x,t) - \Omega e^{-ik_0 z}\} r_3(x,t) \quad (1a)$$

$$\dot{r}_3(x,t) + 2\gamma_0(\frac{1}{2} + r_3(x,t)) = -i\{E^*(x,t) - \Omega e^{-ik_0 z}\} r_-(x,t) + c.c. \quad (1b)$$

with  $\dot{r}_- = (\dot{r}_+(x,t))^*$  (and  $*$  = c.c.):  $\delta = (\omega - \omega_s)\gamma_0^{-1}$  is the scaled detuning from atomic resonance at  $\omega = \omega_s$ ,  $\gamma_0$  is the A-coefficient, and  $\Omega = p\hbar^{-1}E_0$  is the laser Rabi frequency:  $p$  is the atomic dipole matrix element. The integral form for the (interatomic) field is

$$E^*(x,t) = np^2\hbar^{-1} \int_{V-v} d\underline{x}' r_+(x',t) \underline{\hat{u}}\underline{\hat{u}} : \underline{F}^*(x,x';\omega) d\underline{x}' ; \quad (2)$$

$\underline{\hat{u}}$  is the direction of  $p = p\hat{u}$ ; the region  $V$  occupied by atoms,  $n$  per unit volume, has a small sphere volume  $v$  about  $x$  removed because of obvious features of the tensor Green's function of Maxwell's wave equation  $\underline{F}(x,x';\omega) \equiv (\nabla\nabla + k_0^2 \underline{U})e^{ik_0 r} r^{-1}$  ( $r = |x-x'|$ ;  $k_0 = \omega c^{-1}$ ;  $\underline{U}$  = unit tensor).

In the steady state  $r_+$  oscillate at the driving frequency  $\omega$ ; a profound complication (compared with linear refractive index theory [2,4]) is that radiation damping in eqns. (1) (described through the  $\gamma_0$ ) means that propagating fields must damp and that consequently the inversion  $r_3(z)$  (now always  $> -\frac{1}{2}$ ) depends on position  $z$  in the FP cavity (we assume normal incidence and transmission through the cavity  $0 \leq z \leq L$  and  $\underline{\hat{u}}$  is transverse to  $z$ ). This means that after factoring  $e^{+i\omega t}$

$$pr_+(x, \omega) \equiv P(x, \omega) = P_{f1}(z) e^{-imk_0 z} + \Lambda_1 P_{b1}(z) e^{+imk_0 z} \quad (3)$$

in which  $m(\omega)$  is a formal refractive index,  $P_{f,b1}(z)$  are forward and backward (+z and -z) going dipole field envelopes and  $\Lambda_1$  is a reflexion coefficient to be determined.

Despite the form of (3) the ideas of the 'optical extinction theorem' [1,2,3,4] can still be applied. The essence of the theorem is that scattering from each point of the surface of the cavity is exactly sufficient to extinguish the incident (free) field, wave number  $k_0$ , at each and every point inside the cavity and replace it by a field like (3) with wave number  $mk_0 \neq k_0$ . The non-linearity of the present application is contained in the need for the  $P_{f,b1}(z)$  in (3): it complicates the analysis but the 'extinction' aspect of the theorem survives intact [1,2].

The final results of the steady state analysis on these lines are these: the non-linear refractive index  $m(\omega)$  satisfies the dispersion relation of Lorentz-Lorenz form in r.w.a.

$$\frac{m^2-1}{m^2-2} = \frac{(-2R_3) 4\pi n p^2 \hbar^{-1}}{3(\omega_s - \omega + \frac{1}{2} i \gamma_0)} \quad (4)$$

For normal incidence on the FP cavity  $0 \leq z \leq L$  the output field  $E_{out}$  relates to the input  $E_0$  by

$$E_{out} = E_0 \hat{u} \left[ \frac{r_3(0)}{r_3(L)} \right] \left[ \frac{2}{1+m} \right] \left[ \frac{2m}{1+m} \right] (\exp-i(m-1)k_0 L) [1 - \{(m-1)^2/(m+1)^2\} \exp-2imk_0 L]^{-1} \quad (5)$$

Only the  $r_3(0)/r_3(L)$  explicitly reflects the non-linear nature of the problem: in all other respects (5) is exactly the description of the output of the FP cavity in linear theory. However the non-linearity is in  $m(\omega)$  because it depends on the number  $R_3$ . And in order to achieve the simplicity of (5) with (4) we need to choose  $R_3$  so that

$$R_3^{-1} = L^{-1} \int_0^L [r_3(z)]^{-1} dz. \quad (6)$$

Thus  $R_3$  is an average of  $r_3(z)$  — but is not a mean field average. Thus though the results (5) with (4) might be desired they are not obvious.

Still they show that dispersive OB can otherwise be described in the expected way. The non-linearity is in  $m$ , and this detunes the cavity through the  $\exp-2imk_0 L$  in (5). This leads to a multibranched OB curve of the type described e.g. by Ikeda [5] for the ring cavity. Absorptive OB arises at atomic resonance via (4). Since the effects of non-resonant atomic levels are easily added the theory seems potentially quantitative. It is easy to extract the simple ideas on cubic input/output OB curves from these formulae: for small  $n$  one finds

$$|1 - \{(m-1)/(m+1)\}^2 e^{-2imk_0 L}|^2 \approx [1 - \{(m-1)/(m+1)\}^2 e^{CR_3}]^2 + 4|(m-1)/(m+1)|^2 \sin^2(\frac{1}{2}(-R_3\delta)). \quad (7)$$

where  $\phi, \delta$  are scaled cavity and atomic detunings and  $C$  is essentially the usual co-operation number. The output/input curve is therefore multi-branched but becomes a simple (cubic) OB curve for  $\sin^2(\frac{1}{2}(-R_3\delta)) \approx \frac{1}{4}(-R_3\delta)^2$ .

The complication, consequent solely on the actual physical situation is that

$$r_3(z) = -\frac{1}{2} - \frac{1}{2} (r_3(z))^{-1} [e^{-i(m-m^*)k_0 z} |P_f(z)|^2 + e^{i(m-m^*)k_0 z} |P_b(z)|^2 |\Lambda|^2] \quad (8)$$

( $\Lambda \equiv \{(m-1)/(m+1)\} \exp -2imk_0 L$ ). This means we have to have a complicated iterative procedure for solving for the output-input curve. This has been carried through for various values of the parameters and results will be reported.

The steady state analysis so far discards all standing wave effects. We are obliged to extend (3) by harmonic terms  $P_{f,b3}(z, \omega) e^{\pm 3imk_0 z}$ , ..... and introduce reflexion coefficients  $\Lambda_3$ , etc. Similarly  $r_3(z)$  must be expanded in  $\cos v (m+m^*)k_0 z$  and  $\sin v (m+m^*)k_0 z$  ( $v=0,2, \dots$ ). The result is a coupled system for envelope functions  $P_{f,b1}(z)$ ,  $P_{f,b3}(z)$ , ..... and  $r_{30}(z)$ ,  $r_{31}(z)$  ..... which also depends very explicitly on  $m$ . Remarkably (or not!) the extinction still operates: the output/input relation (5) is changed by the harmonics, the significance of the FP action is changed somewhat, but all functions and parameters including  $m$  are determined by the theory which is still derived only from the Maxwell-Bloch equations and the cavity condition that atoms lie only in  $0 \leq z \leq L$ .

Finally we are studying the dynamical stability of the multibranched input/output curve. We expect bifurcations to one or more oscillation frequencies, period doubling sequences and "chaos" of the type predicted and seen [6] for the hybrid device ring type cavity, and we think there are physically realisable parameters which would permit observation of chaos in the FP cavity. Certainly it is possible to reach a non-linear difference equation of the form  $P_f(t+t_R) = A\Omega + BP_f(t) \cdot \exp i(\theta - \phi)$ , where  $A, B$  depend on  $m$ ,  $t_R$  is a double pass path delay in the cavity (governed by a speed  $V \neq c$ ) and  $\theta$  depends on the integral of  $|P_f(t+xt_R)|^2$  ( $0 \leq x \leq 1$ ) across the whole cavity. Firth [7] reported the same (generic) form with a significant difference in the interpretation of the parameters and in related work [8] reports period doublings and apparent chaos.

We shall report the results of our fundamental study of dynamics and "chaos" in the FP cavity at the meeting.

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# Optical Bistability from the Dicke Model

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We have reported a fairly complete analysis of the quantum Dicke model [1]. This consists of  $N$  2-level atoms on the same site driven by arbitrary fields - coherent, chaotic, mixed coherent-chaotic. We also showed [2] in particular that it is easy to include, albeit in a rather formal way, the effects of a cavity on the Dicke model in a broad band chaotic field. The results agree well with those recently observed on high Rydberg Na atoms [3]. This suggests that a comparable analysis for a coherent driving field which also includes the effects of a cavity will provide a quantum theory of optical bistability. It also suggests that some of the consequences of the theory, primarily photon statistical properties, which, however, are calculated as interatomic correlations [2,3], might be observable in a suitable experiment.

The equation of motion for the total density operator  $\rho$  of the system can be taken to be

$$\frac{d\rho}{dt} = -i[H_0 + H_1 + H_2, \rho(t)] + \Lambda_a \rho(t) + \Lambda_f \rho(t) \quad ; \quad (1)$$

$H_0 = \omega_s S_z + \omega_s a^\dagger a$ ;  $H_1 = -gaS_+ + ga^\dagger S_-$ ;  $H_2 = k(a^\dagger E(t) - aE^*(t))$  in which  $S_\pm, S_z$  are the usual collective Dicke operators [1,2];  $g$  is a coupling constant and there is a preferred mode [1,2] on exact resonance with the atomic resonances at  $\omega = \omega_s$ . The operator  $\Lambda_a$  is shown to be such that

$$\Lambda_a \rho = \gamma_0 [2S_- \rho S_+ - S_+ S_- \rho - \rho S_+ S_-] \quad (2)$$

in which  $\gamma_0$  is the A-coefficient; this describes collective spontaneous emission and seems to accord well with the experiments in the incoherent case [2,3]. It differs profoundly from the model of super-radiance [4] with which it might be compared. The damping of the cavity mode is described by  $\Lambda_f$  where

$$\Lambda_f \rho(t) = \kappa [2a \rho a^\dagger - a^\dagger a \rho - \rho a^\dagger a] \quad (3)$$

It is easy to show from (1) that an arbitrary atomic operator  $Q$  and the preferred field mode operator  $a$  satisfy respectively the equations of motion

$$\begin{aligned} \frac{d\langle Q \rangle}{dt} &= +i\omega_s \langle [Q, S_z] \rangle - ig \langle [Q, aS_+] \rangle \\ &\quad + \gamma_0 \langle [2S_+ Q S_- - Q S_+ S_- - S_+ S_- Q] \rangle + ig \langle [a, S_- a^\dagger] \rangle \end{aligned} \quad (4a)$$

$$\frac{d\langle a \rangle}{dt} = i\omega_s \langle a \rangle - \kappa \langle a \rangle + ikE + ig \langle S_- \rangle \quad (4b)$$

We have not (yet) solved this system exactly. But the work on the coherently driven Dicke models [1a] indicates that for large enough  $N$  ( $N \rightarrow \infty$ ) we can de-correlate  $\langle aA \rangle = \langle a \rangle \langle A \rangle$  when  $A$  is an arbitrary atomic operator. De-correlating eqns. (4) this way and moving to a frame rotating at  $\omega_s$ , we find (with  $\alpha \equiv \langle a \rangle$ ) that

$$\frac{d\langle Q \rangle}{dt} = -ig\alpha \langle [Q, S_+] \rangle + ig\alpha^* \langle [Q, S_-] \rangle + \gamma_0 \langle [2S_+QS_- - QS_+S_- - S_+S_-Q] \rangle \quad (5a)$$

$$\frac{d\alpha}{dt} = -\kappa\alpha + ikE + ig\langle S_- \rangle \quad (5b)$$

For a theory of bistability we need the steady state solution of this system. We can use the exact results for the coherently driven Dicke model obtained previously [1]. For  $N \rightarrow \infty$  we know that a solution of (5a) for  $Q \equiv S_-$  can be obtained in the form  $\langle S_- \rangle / N = -\langle S_+ \rangle / N \equiv iS$  with  $S$  given by

$$S = \frac{1}{2}x, \quad x \leq 1; \quad = \frac{1}{2}x[1 - \sqrt{x^2-1}/x^2 \sin^{-1}(1/x)], \quad x \geq 1; \quad (6)$$

where  $x = g(\alpha + \alpha^*)/\gamma_0 N$ . Also the steady state solution of (5b) yields

$$2S = -(\kappa\gamma_0/g^2)(x-y) \quad (7)$$

where  $y \equiv -ig(E-E^*)/\gamma_0 N$  is proportional to the incident field. Thus we find

$$y = (1+C)x, \quad x \leq 1; \quad = x[1+C(1 - \sqrt{x^2-1}/x^2 \sin^{-1}(1/x))] \quad x \geq 1; \quad (8)$$

where the co-operation number  $C \equiv g^2\kappa^{-1}\gamma_0^{-1}$ . The plot of  $x$  against  $y$  for different  $C$  (shown in Fig. 1 for finite  $N = 50$ ) exhibits conventional bistable behaviour for  $C \gtrsim 0.1$ . Still this is  $C \ll 4$  and the curve is not of course the usual cubic catastrophe curve: for  $N \rightarrow \infty$  it has a cusp between the (stable) lower branch and the (unstable) portion of negative slope, though hysteresis must occur in practice. Moreover this quantum theory shows that the intensity-intensity correlations  $g^{(n)}(0) = \langle S_+^n S_-^n \rangle / \langle S_+ S_- \rangle^n$  approach asymptotically the values found previously: in particular  $g^{(2)}(0) = 1.2 < 2$ , so above the phase transition at  $y = 1+C$  the bistable system remains partially coherent.

If, on the other hand, we try to solve eqns. (5) in semiclassical approximation by de-correlating the atomic operators we find that the solution of the resulting equations in the steady state is

$$\langle S_z \rangle = -\frac{1}{2}\sqrt{1-\theta^2}, \quad \theta \leq 1; \quad = 0, \quad \theta \geq 1; \quad (9a)$$

$$\langle S_y \rangle = \frac{1}{2}\theta, \quad \theta \leq 1; \quad = 1/2\theta, \quad \theta \geq 1; \quad (9b)$$

where  $\theta \equiv g|E-E^*|/N(\gamma_0 + g^2\kappa^{-1})$ . These are just the semi-classically approximated solutions of the coherently driven Dicke model (with  $\gamma_0 \rightarrow \gamma_0 + g^2\kappa^{-1}$ ), and there is no bistability.

For  $x < 1$  the resonance fluorescence spectrum is that already calculated [1]: for adiabatic elimination of  $\langle a \rangle = \alpha$  in (5) yields the usual Dicke model equations. These still have not been solved exactly for transients; but for  $N \gg 1$  the semi-classical approximation is rigorously valid. We discussed [1] the possibility of a comparable analysis for  $x \geq 1$ .

The conclusion from this strictly quantal model coupled to a cavity must be that quantum fluctuations and cavity feed back are together sufficient for optical bistability but that either separately is not. But this quantal model is strictly collective: it contrasts with ref. [4] and the more recent work on OB [5] where spontaneous decay is the sum of individual atom decays: thus typically  $C = Ng^2/\kappa\gamma_0$  in Ref. [5] but is  $C = Ng^2/\kappa N\gamma_0$  in our work — which means that  $\gamma_0 \rightarrow N\gamma_0$  as usual. This is consequent on the fact that  $\dot{S}^2$  is a constant of the motion, a feature only broken by de-correlation. On the other hand in extended systems (see e.g. [6]) semi-classical theory, in which  $\dot{S}^2$  is not a constant, gives a good description of optical bi- and multi-stability. The precise significance of quantum fluctuations to OB in general is therefore to be determined.

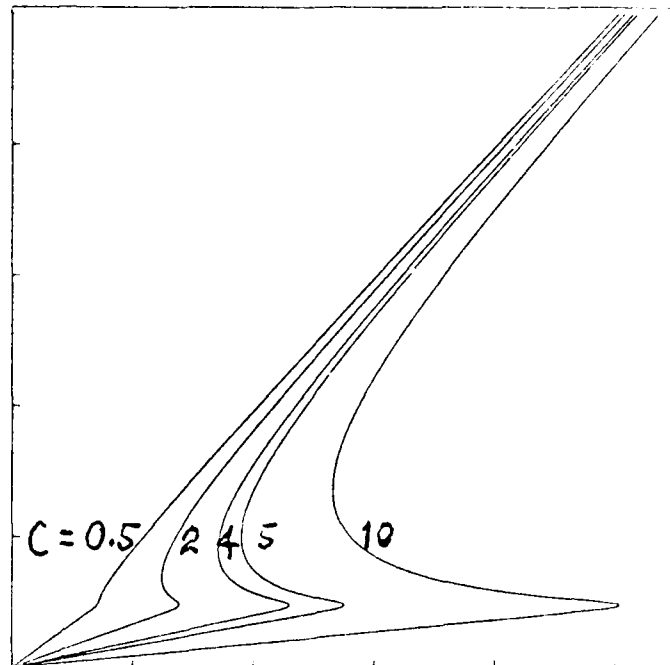


Fig. 1. Plot of  $x$  against  $y$  for  $N = 50$

This paper is an expanded version of the work reported earlier [7]

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NOTES

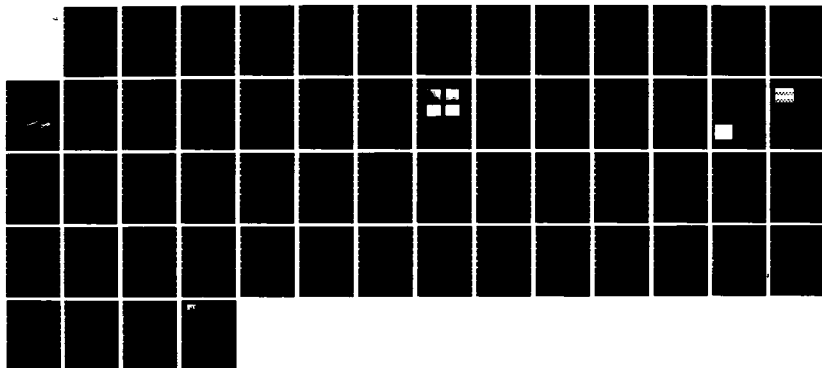


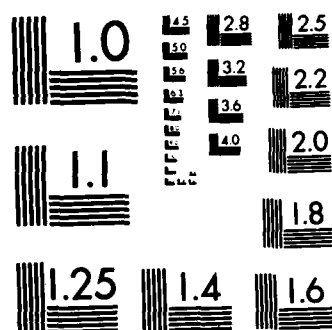
TOPICAL MEETING ON OPTICAL BISTABILITY HELD AT 3/3  
ROCHESTER NEW YORK ON 15-17 JUNE 983(U) OPTICAL SOCIETY  
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# Theory of Resonance-Enhanced Optical Nonlinearities in Semiconductors

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In semiconductors the attractive Coulomb interaction between electrons and holes causes one- and two-photon resonances, known as excitons and exciton molecules, respectively. Large nonlinearities are observed in the spectral vicinity of these resonances. These nonlinearities are well-suited for the realization of optical bistability<sup>1),2),3)</sup> as has been demonstrated by Gibbs et al.<sup>1)</sup> for GaAs and by Miller et al.<sup>2)</sup> for InSb.

The electron-hole correlations, the screening and the energy renormalization are most conveniently calculated by the Green's function formalism. The optical dielectric function can be expressed in terms of the retarded photon self-energy which is also called the polarization function. This connection is not restricted to the linear response regime.

In narrow-gap semiconductors the low-intensity optical spectra in the band-gap region are strongly influenced by the electron-hole correlation. For high intensities, the screening of the Coulomb interactions is so large that these correlations become less important. In this limit, the optical spectra are those of an electron-hole plasma. In the transition region the optical spectra change strongly.

We have been able<sup>4),5)</sup> to develop a calculation of the dielectric function which is valid from the low-intensity excitonic limit up to the quasi-metallic high-intensity limit in which free-carrier screening is the most important feature. We neglected the contribution of the excitons to the screening and used a quasi-static RPA approximation. The integral equation for the complex polarization function has been solved numerically by matrix inversion. The resulting complex dielectric function gives immediately both the absorption and the refraction spectra without using a Kramers-Kronig transformation.

Fig.1 shows the resulting spectra for GaAs for various free-

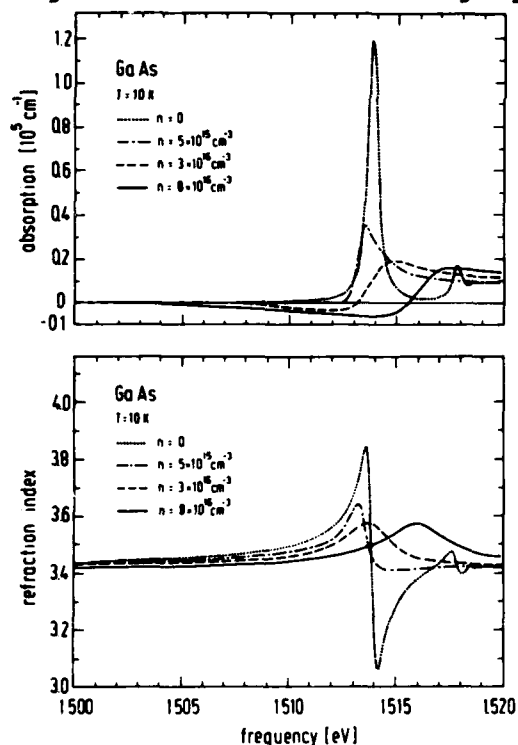


Fig.1: Calculated spectra of absorption and refraction for GaAs at various free-carrier densities and a temperature of 10 K.

carrier densities. This density can be related to the light intensity by a rate equation. For the curve with a free-carrier concentration of  $n=5 \times 10^{16} \text{ cm}^{-3}$  the exciton energy equals the renormalized band-gap, so that the exciton is ionized. In this situation still a considerable excitonic enhancement exists. At higher densities a further band-gap shrinkage and a band-filling is seen, so that optical gain develops for  $\omega < \mu$  where  $\mu$  is

is the quasi-chemical potential. Naturally, in a single-beam experiment  $\mu$  is always smaller than the laser frequency. The refraction index is seen to decrease in the frequency range below the exciton resonance as the exciton is ionized.

The resulting spectra for InSb (Fig.2) show no resolved exciton resonance, because the energy broadening is twice the exciton binding energy. The resonance enhanced absorption edge is seen to be bleached by the increasing light intensity and a corresponding decrease of the refraction index is obtained.

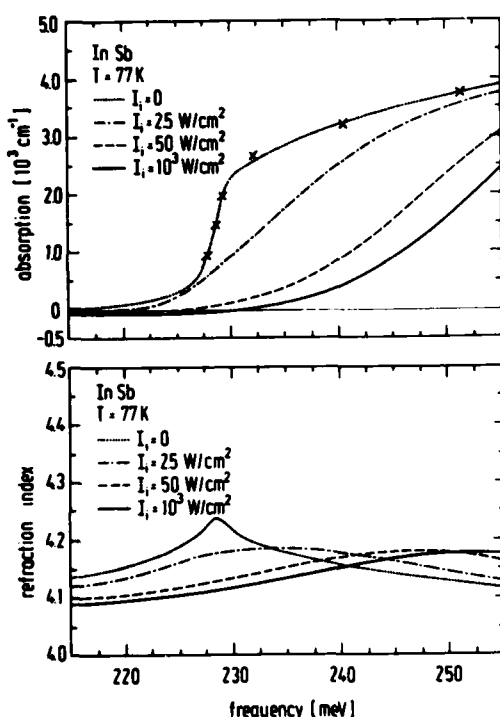
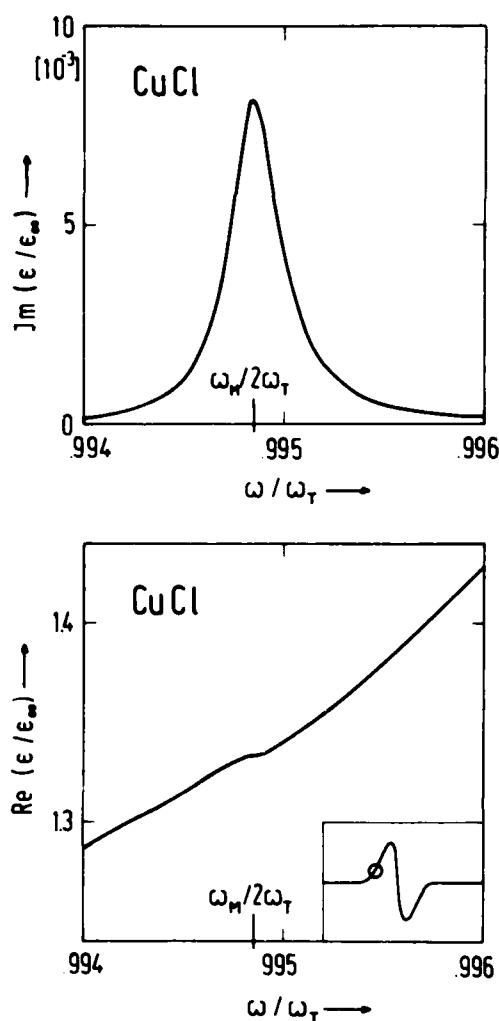


Fig.2: Calculated spectra of absorption and refraction for InSb at various internal excitation intensities  $I$  at 77 K. The excitation laser frequency was assumed to be  $\omega = 225$  meV.

The theoretical results both for GaAs and InSb are at least in good qualitative agreement with all known experimental observations. In wide-gap semiconductors the static dielectric constant of the unexcited crystal is smaller and therefore the exciton-hole correlations are stronger. In cuprous halides, e.g., one observes not only a one-photon excitonic resonance but also a two-photon resonance due to the exciton molecule. The first photon creates a virtual exciton which fuses with the second photon to yield a

real or virtual molecule. The corresponding intensity-depen-



dent photon self-energy can be calculated analytically (Ref.3 and references quoted therein). Fig.3 shows the resulting real and imaginary part of the optical dielectric function for CuCl. The calculated two-photon resonance is in principle strong enough for the realization of optical bistability also with wide-gap semiconductors.

Fig.3: Calculated dielectric function for CuCl at a polarization density of  $4 \times 10^{15} \text{cm}^{-3}$ .  $\omega_T$  and  $\omega_M$  are the frequencies of the transverse exciton and of the molecule, respectively.

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Room Temperature Optical Nonlinearities  
in GaAs Multiple Quantum Wells

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Progress towards practical all-optical switching and signal processing devices depends crucially on finding suitable nonlinear materials. Consequently, various workers have explored so-called 'dynamic' nonlinear optical effects in semiconductors<sup>(1)</sup> in which the optical properties change as a result of real excitation of the material. Semiconductors are particularly attractive for fabrication as small devices. Unfortunately, while some of these effects should persist to room temperature, one of the most attractive, saturation of free exciton resonances<sup>(2)</sup> is basically limited to low temperatures in conventional semiconductors because of thermal broadening.

By confining carriers in GaAs quantum wells  $\sim 100\text{\AA}$  thick, spaced apart with GaAlAs barriers, exciton resonances can be observed at room temperature,<sup>(3)</sup> primarily because the confinement leads to an increase in binding energy,  $\sim x2$  for  $100\text{\AA}$  layers, whereas in GaAs the resonance has all but disappeared at room temperature. Furthermore, the MQW

absorption near the exciton peaks is much more readily saturated than GaAs near its exciton peak at room temperature.<sup>(3)</sup>

The mechanism behind the excitonic absorption saturation at room temperature is thought to be as follows.<sup>(3)</sup> Optical absorption near the resonance creates excitons; within  $\sim 0.5$  ps these excitons are ionized into free electrons and holes. This free carrier density interferes with the production of further excitons, thus altering the excitonic absorption spectrum. The free carriers (and consequently the changes in absorption and refraction) decay with  $\sim 20$  ns lifetime.

These observations have stimulated renewed interest in MQW structures. Optical bistability has been observed in room-temperature MQW structures,<sup>(4)</sup> as has degenerate four-wave mixing (DFWM).<sup>(5)</sup> Optical bistability observations have so far greatly exceeded the measured excitonic saturation intensities and may have to rely on other processes (e.g., interband saturation) to complete the switching action.

Unfortunately the microscopic theory<sup>(3)</sup> is currently too crude to be of much help in designing efficient devices. Detailed measurements of low intensity behavior<sup>(5)</sup> have however recently been made.<sup>(5)</sup> Single laser



nonlinear absorption spectra show that the exciton peaks initially broaden without loss of area and shift to lower energy. Recent experiments using two lasers support this interpretation. DFWM spectra combined with the nonlinear absorption can be used to measure nonlinear refraction, giving  $\Delta n/N \sim 2 \times 10^{-19} \text{cm}^3$  where  $\Delta n$  is the change in refractive index and  $N$  is the induced carrier density. This is relatively a very large number, and if devices can be designed to utilize such nonlinearities operating powers may be reduced by orders of magnitude.

Work is continuing to characterize these nonlinearities in more detail, and currently these materials offer good prospects for practical room-temperature all-optical switching and signal processing devices compatible with laser diodes.

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Nonlinear Optical Response of the Lowest Exciton Resonances in GaAs

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# NOTES

## Considerations Involved in the Design of an Optical Digital Computer

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### ABSTRACT

The prime concern in the design of an optical digital computer is "Why?". Optical bistability is faced with the same question. If optical bistability is to compete with other switching technologies solely in terms of speed and power then its development will be haunted by the constant advances in silicon technology.

Optical bistability is different from other switching technologies in that it is based on optics rather than electronics. This would seem to be a disadvantage since it would be more difficult to integrate such devices into an electronic design, however there are advantages. One advantage is the parallelism of optics while another is its non-interfering propagation. A lens or a mirror can easily communicate thousands of channels of information in parallel. One optical beam can also be passed through another without interference. Electronic signals on the other hand must be individually guided and protected from interacting.

It is difficult to appreciate these advantages until one studies some of the problems confronting future computer systems. One problem involves interconnection bandwidth. As the switching speed increases, more bandwidth is needed to communicate. This forces the use of microstrip transmission lines and terminated coaxial interconnections. These faster switching speeds also aggravate the problem of clock skew. This limits the maximum interconnection length and also limits the maximum difference in interconnection lengths. The problems of bandwidth and clock skew combine to restrict the topology and greatly complicate the design of future systems.

An even more fundamental problem is that of the Von Neumann bottleneck. This flaw is exemplified by the serial, address laden communication which permeates all current computers.

This bottleneck can be traced to a tradeoff of time for interconnections which is needed to compensate for the inability of electronics to support  $n$  communication channels in parallel. One result of this bottleneck is that current processors can only manipulate a very small amount of its state space during each cycle. This means it takes more time to perform a given task.

It is well known that optics can provide many high bandwidth equal-distant interconnections in parallel. The question is how to modify the architecture of current computers to exploit these capabilities. An optical digital computer based on a parallel finite state machine is used to illustrate how this might be accomplished. This processor consists of a two dimensional array of optical NOR gates, prisms and lenses capable of offsetting the output image of the NOR gate array one pixel in each direction, several latches capable of storing the offset image of the NOR gate array, and a means of feeding the output image of the latches to the input of the NOR gate array. This design is used to demonstrate how the problems of bandwidth, clock skew, and the Von Neumann bottleneck can be avoided by using optics. This design is also used to illustrate how optical devices can be integrated into a computational structure and how the capabilities of optics can be used to give optical bistability an edge over other switching technologies.

Polarization Switching with Sodium Vapour  
in a Fabry-Perot

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We report results of an experiment which demonstrates switching of the polarization state of the output of a near-concentric Fabry-Perot containing sodium vapour, excited via the *homogeneously* broadened D1 transition. An example of the output, for linearly polarized laser input, at 1.5 GHz below line centre (illustrated in figure 1) shows switching with hysteresis at high input power ( $\sim 400$  mW) due to saturation, as well as the switching at lower power ( $\sim 200$  mW) due to optical pumping effects.

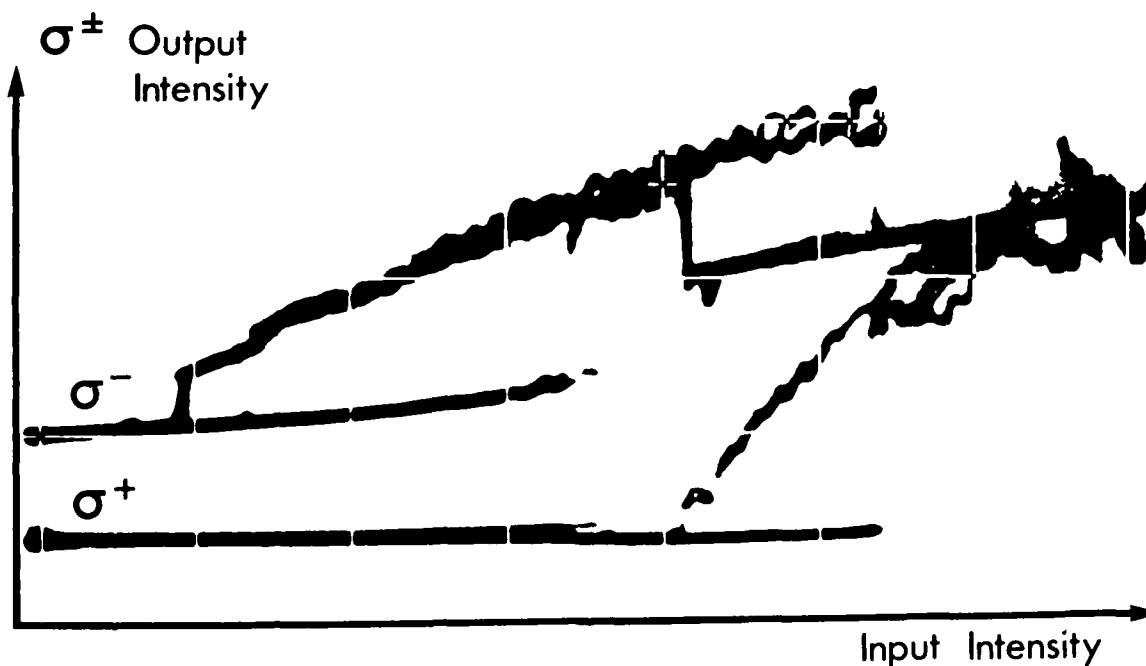


Figure 1.

We have also observed purely absorptive polarization switching at ~50 mW input power.

Polarization switching has been predicted in the context of three state atoms in an optical cavity<sup>1</sup>, and the lower power switching has been previously observed using the inhomogeneously broadened D1 transition of sodium in a Fabry-Perot with the laser detuned above line centre<sup>2</sup>.

By using argon buffer gas at up to 100 torr, we have reduced the importance of hyperfine structure and inhomogeneous broadening so as to make the transition approximate a  $J = \frac{1}{2}$  to  $J = \frac{1}{2}$  transition<sup>3</sup>. The behaviour of the switchings that we observe will be compared with a theory based upon  $J = \frac{1}{2}$  to  $J = \frac{1}{2}$  transition including the effect of magnetic fields<sup>4</sup>.

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and to be published.



**Transverse Solitary Waves in a Dispersive Ring Bistable Cavity  
Containing a Saturable Nonlinearity**

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Stationary and nonstationary solitary waves can gradually evolve when the incident Gaussian beam intensity exceeds threshold for switching to the high transmission branch of a ring bistable cavity. The number of transverse solitary waves scales as the square root of effective Fresnel number and as the incident laser amplitude.

At high Fresnel number, the switching of an incident Gaussian pulse to the high transmission branch of a ring bistable cavity exhibits strong radial dependence. The portion of the beam profile satisfying  $I(x) > I_{\text{th}}$  ( $I(x)$  is the intensity in the transverse  $x$ -co-ordinate and  $I_{\text{th}}$  is the plane wave predicted switch-on intensity) switches rapidly to the high transmission branch in about 20 cavity roundtrips. The central portion of the beam on recovering from the overshoot switch causes a singular type edge to appear at the outer extremities of the on-spot. The subsequent dynamics is dependent upon whether the system is operating under self-focusing or defocusing conditions.<sup>1</sup>

In the good cavity limit, the wave equation governing propagation through the nonlinear medium may be written

$$\frac{\partial E(x,y,z)}{\partial z} = -\frac{\omega_0}{2} \left[ \frac{\Delta - i}{1 + \Delta^2 + |E|^2} - \frac{i\ell n_2}{2\pi\alpha_0 L_F} \nabla_{\text{t}}^2 \right] E(x,y,z) \quad (1)$$

and further, the transverse  $E$  field satisfies the following boundary conditions.

$$E(x,y,0,t) = \sqrt{T} E_{in}(x,y,0,t) + \text{Re}^{ik\mathcal{L}} E(x,y,L,t-l/c) \quad (2)$$

The first term on the right hand side of Eq. (1) is the saturable term which gives rise to self-focusing or self-defocusing while the last term is responsible for diffraction [ $\nabla_t^2 = w_0^2 (\partial^2 / \partial x^2)$ ; we assume a single transverse dimension];  $\Delta = (\omega - \omega_{ab}) / \gamma_1$  is the normalized laser atom detuning,  $\alpha_0$  the linear absorption coefficient per unit length,  $F = n_0 w_1^2 / \lambda L$  is the Fresnel number ( $n_0$  the background refractive index,  $w_1/2$  is the beam waist half-maximum,  $\lambda$  wavelength and  $L$  the nonlinear medium length;  $\mathcal{L} = L+l$  is the total cavity length).

At high Fresnel number in conventional propagation problems it is customary to drop the Laplacian term and adopt the so called uniform plane wave approximation. Optical bistable switching is unique however in that the singular edge discussed above causes the Laplacian to be important locally and when operating under self-focusing conditions a train of solitary waves begins to evolve from both outer edges towards the center of the beam. The train of solitary waves can be stationary or can undergo complicated periodic oscillations in the transverse dimension depending on the total area of the switched-on beam. Figure 1 shows the initial stage of the switching process to the high transmission branch for a Fresnel number of (a)  $F = 50$  and (b)  $F = 2200$ . Once the singular type edge has appeared the solitary waves begin to evolve inwards. Figure 2 shows a self-sustained transverse oscillation which repeats after approximately 85 cavity roundtrips ( $F=50$ ).

A soliton theory has been developed, which replaces the complicated numerical problem of solving both the nonlinear partial differential equation (propagation) and eqn(2) over many hundreds of cavity roundtrips by a solution of two ordinary differential equations and a few simple projections of the results. Quantitative predictions can be made for the shapes, widths and heights of the solitary soliton trains. The prediction of the theory is exact for a single soliton appearing on the upper branch in the bistable region.

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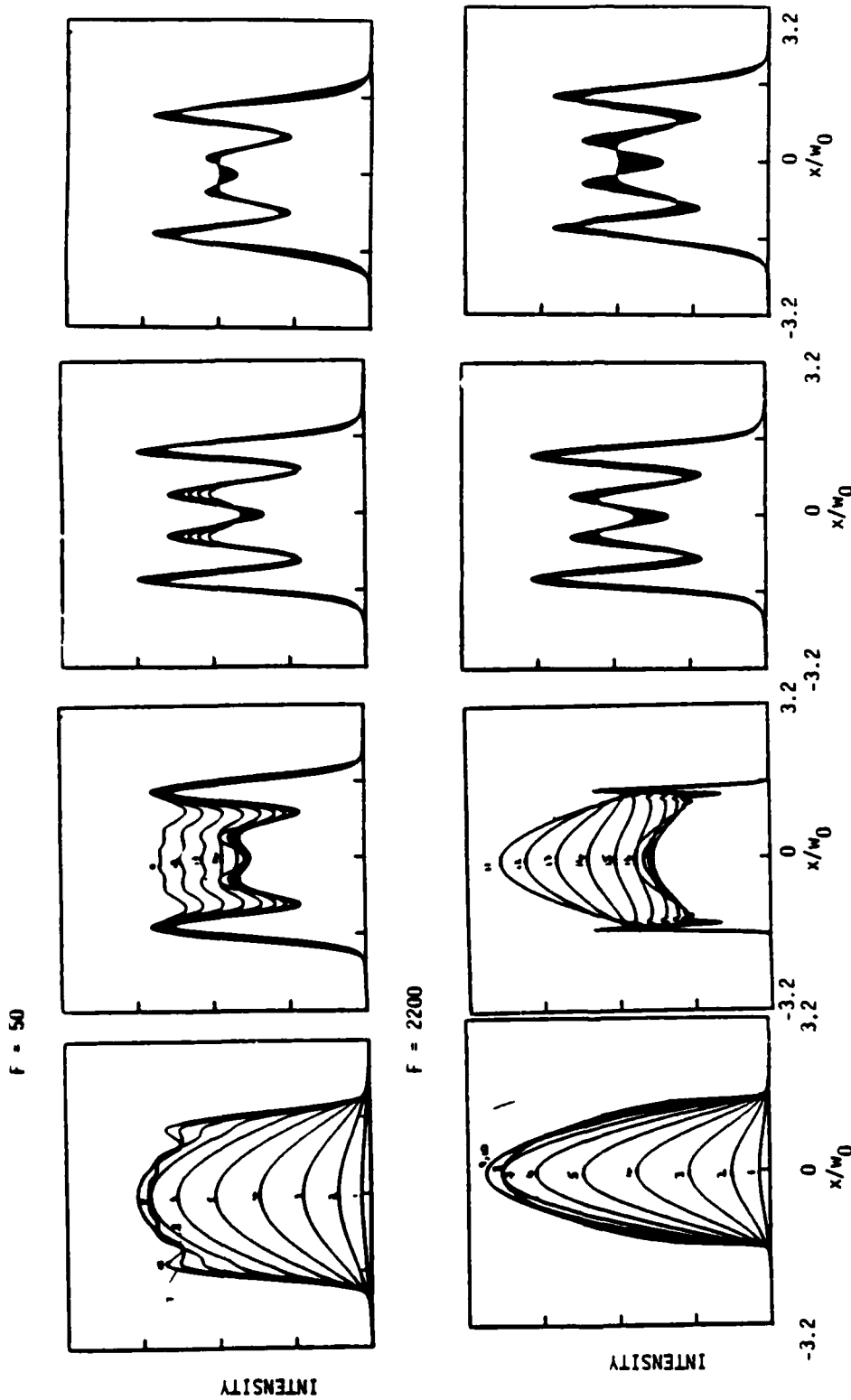


Figure 1. Transient buildup of the intracavity transverse profile on successive passes for Fresnel numbers  $F=50$  and  $F=2200$ . (a) switch up on the first 10 passes and (b) recovery from overshoot switch and solitary wave train initiation.

Figure 2. Periodic solitary wave train for  $F=50$ . This figure displays successive transverse outputs beginning at pass 21 and ending at pass 90. The pattern repeats itself approximately every 85 passes.

## Observation of Optical Hysteresis in an all-optical passive ring cavity

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We report the first observation of dispersive optical hysteresis in an all optical passive uni-directional ring cavity system. Ammonia gas is used as the non-linear medium for which a number of vibrational-rotational transitions<sup>(1)</sup> are in near coincidence with the step tunable emission from a TEA CO<sub>2</sub> laser. Pronounced instabilities and optical hysteresis are found to occur over a gas pressure range commensurate with that for self-focussing. A grating tunable hybrid TEA CO<sub>2</sub> laser with amplifier, operating on a single mode, with a gain switched pulse duration of  $\sim 100$  nsec was used. The laser output, sampled via a beam splitter, was coupled through a Ge flat ( $R = 36\%$ ) which formed one element of the three element ring cavity (cavity length  $\sim 3.5$  m) comprising two plane mirrors and incorporating a 1 m long gas cell and beam splitter for sampling the cavity signal. Signals were recorded using photon drag detectors in conjunction with Tektronix 7904/7104 oscilloscopes.

Typical results, shown in Fig. 1 for an input signal  $\sim 400$  kW, were obtained for off-resonant excitation of the  $aR(11)$  transition of NH<sub>3</sub> with the  $10R(14)$  line of the CO<sub>2</sub> laser ( $\nu_{\text{CO}_2} - \nu_{\text{NH}_3} = +1.23$  GHz). For the pressure range (9-15 torr) over which instabilities were found, the pressure broadened bandwidth of the transition<sup>(2)</sup> ( $\sim 13$  MHz/torr) is considerably smaller than the frequency mismatch of 1.23 GHz, indicating the dominant role of dispersion in these effects.

Oscilloscope traces (a) and (b) are simultaneous recordings of the input signal and the ring cavity signal for an NH<sub>3</sub> pressure of 15 torr.

The temporal features of the cavity signal was sensitive to gas pressure. Outside the pressure range for instabilities (and self focussing) the ring signal exhibited a smooth temporal form similar to that of the incident signal. Cavity blocking and cavity misalignment checks unambiguously identified these nonlinear effects to be due to cavity feedback. Traces (c) and (d) are dynamic plots of the input signal (abscissa) versus ring cavity signal (ordinate) obtained by recording the respective signals on the x and y plates respectively of a Tektronix 7104 oscilloscope. The linear dependence of trace (c) for zero  $\text{NH}_3$  pressure serves as a normalisation curve. Hysteresis associated with optical bistability is clearly evident in trace (d) for an  $\text{NH}_3$  pressure of 10 torr, and is characteristic of results obtained over the pressure range indicated above.

We will report new experiments and theoretical modelling aimed at characterisation of these effects and at observation of Ikeda instabilities<sup>(3)</sup> and chaos in all optical systems of this type.

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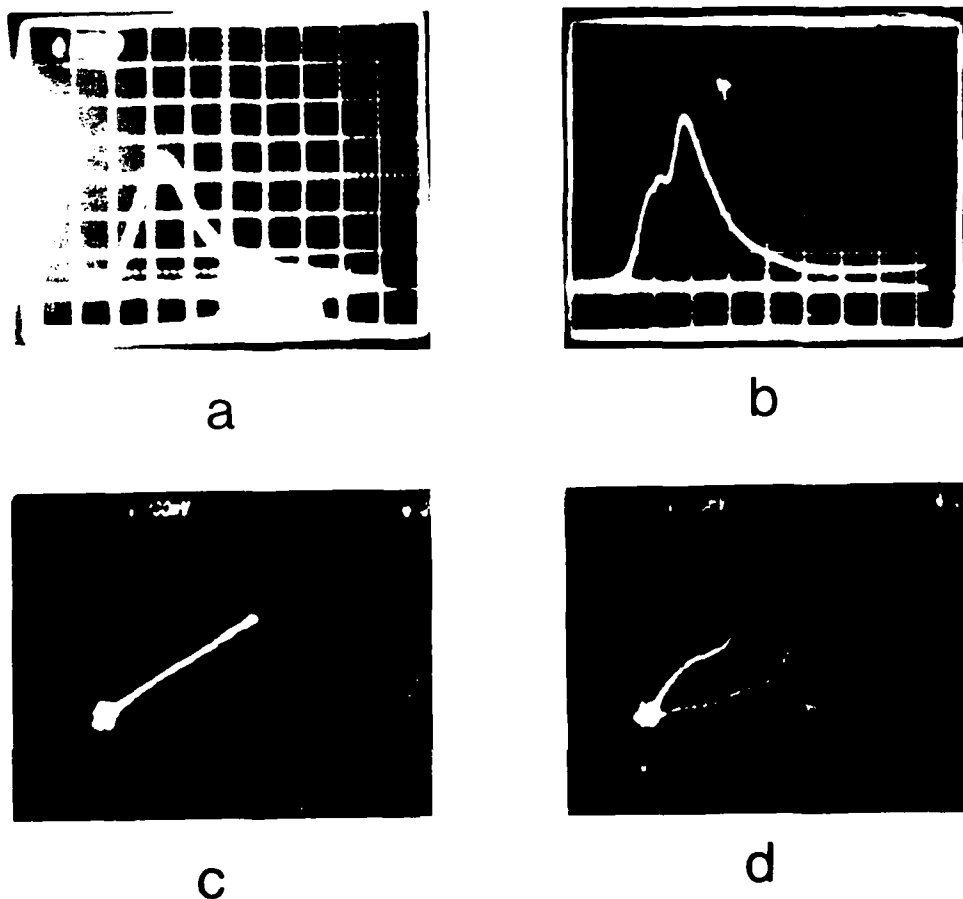


Figure 1 Traces (a) and (b) are of the TEA  $\text{CO}_2$  laser input signal and ring cavity signal respectively (pulse duration  $\sim 100$  nsec  $\text{NH}_3$  pressure 15 torr). Traces (c) and (d) for  $\text{NH}_3$  pressures of zero and 10 torr respectively are dynamic recordings (see text) of input versus ring cavity (ordinate) instantaneous intensities.

Absorptive Optical Bistability in a bad cavity.

Paul Mandel

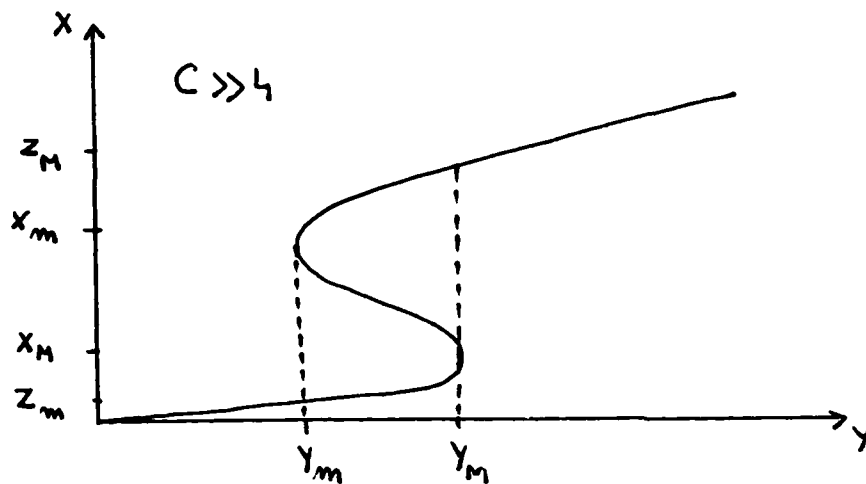
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In a theoretical discussion of optical bistability, it is often necessary to resort to some approximation scheme to discuss time-dependent properties. The standard method consists in assuming a very good cavity ( $K \ll \gamma_L, \gamma_u$ ) or a very bad cavity ( $\gamma_u \ll K, \gamma_L$ ) and to perform an adiabatic elimination of the fast varying variables. In the good cavity limit /1/ the atomic variables are eliminated and one is left with an equation for the field. In the bad cavity limit /2/ the field and the atomic polarization are eliminated and one is left with an equation for the atomic inversion.

We show that when all three decay constants  $K, \gamma_L$  and  $\gamma_u$  are of the same order of magnitude an adiabatic elimination of fast variables can nevertheless be performed in the limit of large  $C$  (fully developed hysteresis). Our method is based on two arguments. First we consider four domains, each centered around one of the four particular points of coordinates  $(Y_M, X_M)$ ,  $(Y_P, Z_P)$ ,  $(Y_m, X_m)$  and  $(Y_m, Z_m)$ .



$$Y_M \sim C, X_M \sim 1$$

$$Y_m \sim \sqrt{8C}, X_m \sim \sqrt{2C}$$

$$Z_M \sim C, Z_m \sim \sqrt{\frac{2}{C}}$$

In each domain all stationary field and atomic functions have a well-defined dependence on  $C$ . We seek time-dependent solutions which have the same scaling. Second in each domain we perform a multiple-time perturbation expansion after the scaling laws have been applied. As a result we obtain in the two domains pertaining to the lower branch a closed nonlinear differential equation for the population inversion which is closely related to the equation derived in /2/. On the other hand we obtain a closed nonlinear differential equation for the field in the two domains of the upper branch; this equation is closely related to the equation derived in /1/.

In the domain surrounding  $(Y_M, X_M)$  the atomic inversion verifies the equation

$$\dot{\delta} = 1 - \delta - \frac{1}{4\delta} \left( Y^2 + \frac{Y_u}{Y_L} Y \dot{Y} \right)$$

where  $Y$  is the (time-dependent) input field amplitude. This equation yields relevant informations on the jump to the upper branch. It has been analyzed, both analytically and numerically, for constant ( $\dot{Y}=0$ ), slowly varying ( $\dot{Y} \ll 1$ ) and quickly varying ( $\dot{Y} \gg 1$ ) input fields. This leads to the



realization that under time-dependent input fields, a dynamical hysteresis occurs which can be very different from the stationary hysteresis. This dynamical hysteresis is shown to be a function of the sweeping velocity of the input field, the initial conditions and the atomic parameters. Particular attention is devoted to the sawtooth pulse.

In the domain surrounding  $(Y_m, X_m)$  the output field verifies the equation

$$\dot{X} = -X + Y - 2/X$$

Here again a dynamical hysteresis is displayed whose properties are mainly related to the sweeping velocity of the input field.

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# CRITICAL SLOWING DOWN AND MAGNETICALLY-INDUCED SELF-PULSING IN A SODIUM-FILLED FABRY-PEROT RESONATOR

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We report on the observation of critical slowing down and of self-pulsing in a Fabry-Perot resonator (FP) filled with sodium atoms under conditions of transverse optical pumping /1/. The experimental set-up (see Fig.1) is similar to the one used in steady-state bistability studies reported before /2/. The dye laser, optically isolated from the FP by means of a Faraday rotator (FR) is detuned from the Na  $D_1$ -line by 10 to 20 GHz. The sodium sample ( $N \approx 10^{13} \text{ cm}^{-3}$ , buffer gas: argon,  $p=200 \text{ mbar}$ ) is placed in a near confocal Fabry-Perot (Beam waist:  $120 \mu\text{m}$ ) and can be subjected to a transverse magnetic field  $B$ . The intensity of the laser beam can be switched by means of an electro-optic modulator (EOM), thus providing input-steps of variable height (power: 5 - 50 mW). A polarizer  $P_1$  is used to either circularly or linearly polarize the input beam. The right- and left circular polarization components of the resonator output beam can be separated by means of a  $\lambda/4$ -plate and a polarizing beam splitter  $P_2$ . A pair of photodiodes  $PD_1$  is used for detection.

With circularly polarized input light optical bistability can be detected by one of the photodiodes. In Fig. 2 the switching delay  $\tau_D$  between switching of the input intensity from 0 to a finite value  $I_{in}$  and switching of the output intensity is shown as a function of

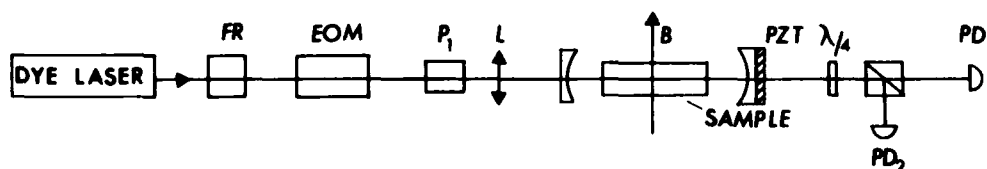


Fig.1

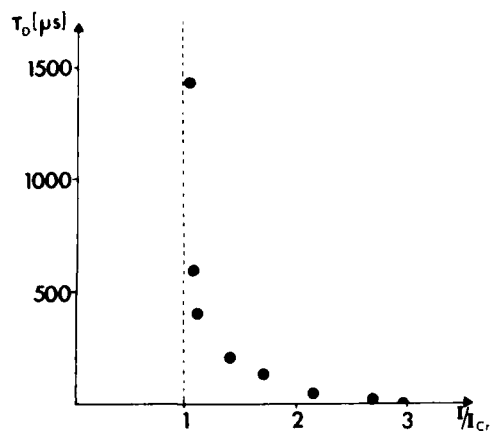


Fig. 2

$I_{in}/I_{cr}$ ,  $I_{cr}$  being the critical input intensity for switching. Obviously  $\tau_D$  displays the phenomenon of critical slowing down. The actual behaviour of the system is described in all details by a simple computer model.

With  $P_1$  being replaced by a linear polarizer oriented not parallel with respect to the magnetic field  $B$ , optical tristability is expected, if the transverse magnetic field is weak /3,4/. (Very recently stationary optical tristability was observed in a longitudinal magnetic field /5/). Fig. 3 shows the time behaviour of the two  $\sigma$ -components following an input step of linearly polarized light with  $I_{in}$  closely above  $I_{cr}$ . After a certain delay  $\tau_D$  the  $\sigma^+$ -component (upper trace) jumps to the lower branch of the tristability curve (see Fig. 2 of Ref. 3), whereas the  $\sigma^-$ -component (lower trace) jumps to the upper branch. Under appropriate conditions coherent Raman-type oscillatory transients are observed in response to bistable or tristable switching.

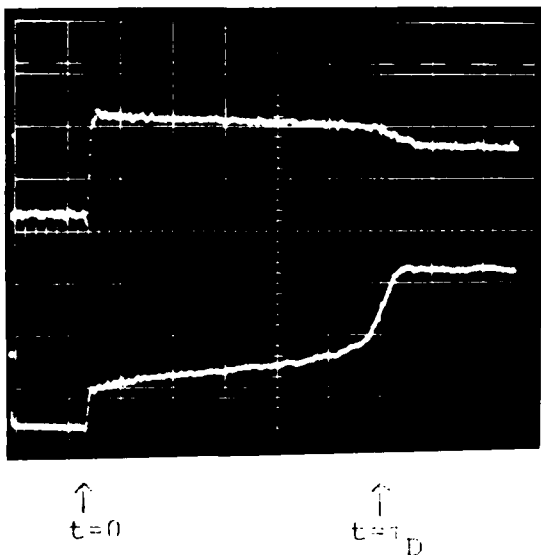


Fig. 3: Transients in optical tristability (10  $\mu sec/div.$ ; upper trace: sensitivity  $\times 2.5$ )

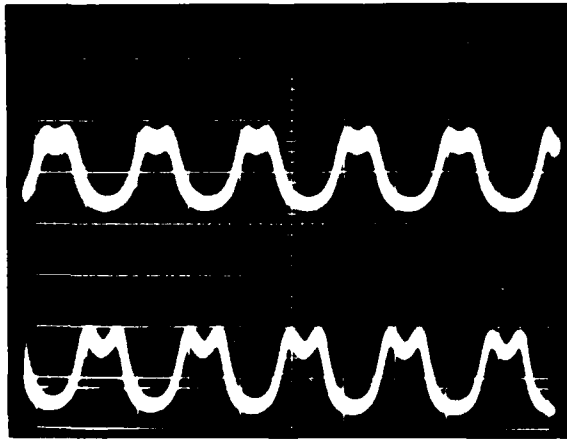


Fig. 4: Magnetically-induced self-pulsation (200μsec/div.)

When the static transverse magnetic field is increased beyond a critical value  $B_{cr}$ , the system starts to switch between the two high-transmission states ( $\sigma^-$ - or  $\sigma^+$ -light transmitted). The intensities of the two polarization components develop into "complementary" regular pulse trains as shown in Fig.4. These pulse trains last for minutes. All observations are in excellent agreement with the predictions of Kitano et al./4/ concerning "magnetically-induced self-pulsing" in optical tristability.

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# Optical Bistability and Hysteresis under Phase Conjugation in CdS Crystal

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## Summary

The exact solution of the nonlinear equations, describing phase conjugation (PC) was obtained. Theoretical dependence of the backward wave intensity on that of interacting waves with Gaussian time shape exhibits well-pronounced hysteresis, which causes the bistable operation when total pump intensity exceeds some critical value  $I_{cr}$ . Several hysteresis loops are expected to appear with further increase of pumping.

The experiments were performed on CdS crystal in four-wave-mixing configuration at room temperature with ruby laser. The onset of bistability was observed at pump intensity above  $20 \text{ MW/cm}^2$ . The modulation of PC intensity due to optical hysteresis was observed. The PC in CdS crystal is due to fifth-order nonlinear susceptibility. This nonlinearity arises from non-equilibrium free carriers excited by interband two-quantum absorption. Its relaxation time is of the order of  $10^{-9} \text{ s}$ .

The characteristic curve of the bistable "mirror" with PC was obtained. Both the value of  $I_{cr}$  and time behavior of the reversed wave are in good agreement with the theory.

The physical nature of the optical bistability and optical feedback in PC under the degenerate four-wave-mixing is discussed. The nonlinear phase mismatch of the interacting waves is found to provide for optical feedback in the scheme in question. The optical feedback in turn leads to hysteretic dependence of the reversed wave intensity versus pump intensity so that optical bistability appears.



LOW POWER OPTICAL BISTABILITY NEAR  
BOUND EXCITONS IN CADMIUM SULFIDE

by

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The  $I_2$  bound exciton in cadmium sulfide has a giant oscillator strength of 6. It decays mostly radiatively in 500 psec. Such a large oscillator strength leads to a very large transition dipole moment and to very large nonlinear effects off-resonance. At 2K, this system saturates like an inhomogeneously broadened system for intensities up to 30 times the saturation intensity. The transition linewidth is 7.5 GHz. In a tight focusing geometry, only 3.6  $\mu\text{W}$  is required to see the onset of saturation. This corresponds to a measured saturation intensity of about 60  $\text{W}/\text{cm}^2$ . The linear absorption coefficient right at the resonance peak was measured to be  $3.2 \times 10^3 \text{ cm}^{-1}$ . At higher temperatures, the transition linewidth broadens and the line becomes more and more homogeneously broadened because of phonon interaction. At intensities above 50 times the saturation intensity, the absorption coefficient stops decreasing and then starts increasing. At these high intensities, thermal effects appear to play a significant role in shifting the free and bound

exciton resonances toward lower energies. At intensities less than 20 times the saturation intensity, thermal effects are believed to be insignificant in our saturation measurements.

For studying optical bistability, the laser beam was focused to a  $20\text{ }\mu$  diameter spot size ( $1/e^2$ ) in a high quality plane parallel cadmium sulfide platelet of  $20\text{ }\mu$  thickness. No reflective coating was deposited on the sample since the discontinuity of the index of refraction is sufficiently large ( $n_{\text{CdS}} \approx 3$ ) for providing the necessary optical feedback. At very low intensities, the far field profile of the output beam from the sample was observed to be Gaussian. As the intensity is increased, the beam diameter increases and a circular fringe pattern develops. This effect is enhanced when the laser is tuned near the exciton resonance. By further increasing the input field, the circular fringe pattern moves in continuously. This happens up to a point when there is a sudden discontinuity in the transmission pattern. By monitoring the intensity of the central fringe only, using an aperture and a detector in the far field, it is found that the transmitted intensity through the aperture increases proportionally with the input intensity to a point where it suddenly drops. Reducing the input intensity, the transmitted light does not follow the same path, but switches back to its initial value at a much reduced input intensity. Very large hysteresis loops are seen and more than one hysteresis loop can be observed depending on the detuning. The appearance of

rings is a clear demonstration of the importance of transverse effects in optical bistability. At high input intensities, very regular pulsations at 56 and 112 KHz were observed as the intensity was raised. There is a regime of input intensities for which the output appears erratic in time.

Using a two-level representation for modeling the material inside a Fabry-Perot resonator, and assuming reasonable physical parameters, we can show that one needs to saturate the transition in order to see optical bistability. This implies that we cannot expect to switch off much faster than the two-level energy relaxation time, given that the cavity build-up time is sufficiently short. On the basis of such a model, we predict a turn-off time from the on state of the order of 500 psec if, of course, thermal effects can be completely eliminated.

In conclusion, optical bistability near a bound exciton in cadmium sulfide can be seen at very low power. Furthermore, this system promises very fast switching and very fast all-optical processing of signals.

# Light-Induced Nonreciprocity: Directional Optical Bistability; Spectroscopy of Induced Anisotropy; Nonlinear Sagnac Effect

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In our recent research<sup>1</sup>, it was suggested that counter-propagating laser beams in a Kerr-nonlinear medium can induce intensity-dependent nonreciprocity which consists in different changes of refractive indices for both of the waves. Under some conditions, this can result in such directionally asymmetrical effects as (i) directional bistability in a nonlinear ring resonator which allows one to switch the direction of propagation of a dominant mode and (ii) enhancement of the Sagnac effect in a rotating ring resonator. Recently, the nonlinear nonreciprocity has been observed experimentally<sup>2</sup>. Due to formation of a nonlinear index grating in the nonlinear medium, the effective susceptibilities for each of the respective waves are

$$\Delta\epsilon_{1,2} = \epsilon_2(|E_{1,2}|^2 + 2|E_{2,1}|^2), \quad (1)$$

where  $\epsilon_2$  nonlinear parameter of the material and  $|E_{1,2}|^2$  are intensities of the waves, such that  $\Delta\epsilon_1 - \Delta\epsilon_2 = \epsilon_2(|E_2|^2 - |E_1|^2)$ . Let a nonlinear ring resonator be pumped in both directions by two incident beams with the same frequency  $\omega$  (in the vicinity of eigen-frequency of the resonator  $\omega_0$ ) and intensity  $|E_{in}|^2$ . Dimensionless intensities of both waves  $I_{1,2}(\propto \epsilon_2|E_{1,2}|^2)$  are then determined by the equation

$$A = I_j\{1 + [\Delta + I_j + 2I_{3-j}]^2\}; \quad j = 1,2 \quad (2)$$

where  $A \propto \epsilon_2|E_{in}|^2$  - dimensionless intensity of the pumping,  $\Delta = \frac{\omega - \omega_0}{\gamma}$  - dimension-

less frequency detuning from the resonance ( $\gamma$  - cavity bandwidth). Eqs(2) always have a symmetric solution ( $I_1 = I_2$ ). However, under some critical conditions, this solution becomes unstable. The new stable regime corresponds to non-symmetrical oscillation : the wave propagating in one direction is suppressing the wave propagating in opposite direction. The dominant wave can be by many orders of magnitude larger than the another one (Fig.1). It could be either left- or right-propagating wave depending on the initial conditions, such that the system exhibits a directional bistability. Inside of the principal domain of a stable symmetrical solution, the system can exhibit an enhancement of the Sagnac effect (the latter is due to linear nonreciprocity originated by the rotation of the ring resonator). The nonlinear Sagnac effect becomes extremely large (with the enhancement of  $10^3 - 10^4$ ) at the onset of the instability and directional bistability. A "hybrid" analog of this system has been also proposed recently<sup>3</sup>.

In this paper, we show that the effect of nonlinear nonreciprocity is directly related to the anisotropic properties of the third-order nonlinear susceptibility. Therefore, it must be extremely sensitive to the mutual arrangement of polarization of both of the counter-propagating waves. In the case of arbitrary third-order nonlinear tensor, the induced anisotropic response of the matter can be expressed as<sup>4</sup>

$$\bar{D}^{NL} = \epsilon_2 \left[ \alpha \bar{E}(\bar{E}\bar{E}^*) + \beta \bar{E}^*(\bar{E}\bar{E}) \right], \quad (3)$$

where  $\alpha$  and  $\beta$  - some constants,  $\bar{E}$  - the total field of both of the waves. Basing on Maxwell equations and Eq(3), we show that these equations have four invariants which allow for some mutual "eigen-polarizations" to propagate without distortion of their initial directions of polarization: both of the beams are linearly polarized, with their electric vectors either (i) parallel to each other or (ii) orthogonal to each other; both of the beams are circularly polarized being either (iii) co-rotating or (iv) counter-rotating. All of those "eigen-polarizations" have different respective magnitudes of nonlinear nonreciprocity. For instance, the ratio of pertinent nonlinear nonreciprocities for these

configurations in the case of pure Kerr-effect is  $1:(-3/4):(1/4):(3/2)$ . Therefore, the proposed effect can be used as a novel experimental tool for the spectroscopy of the anisotropic components of the third-order nonlinear tensor of matter.

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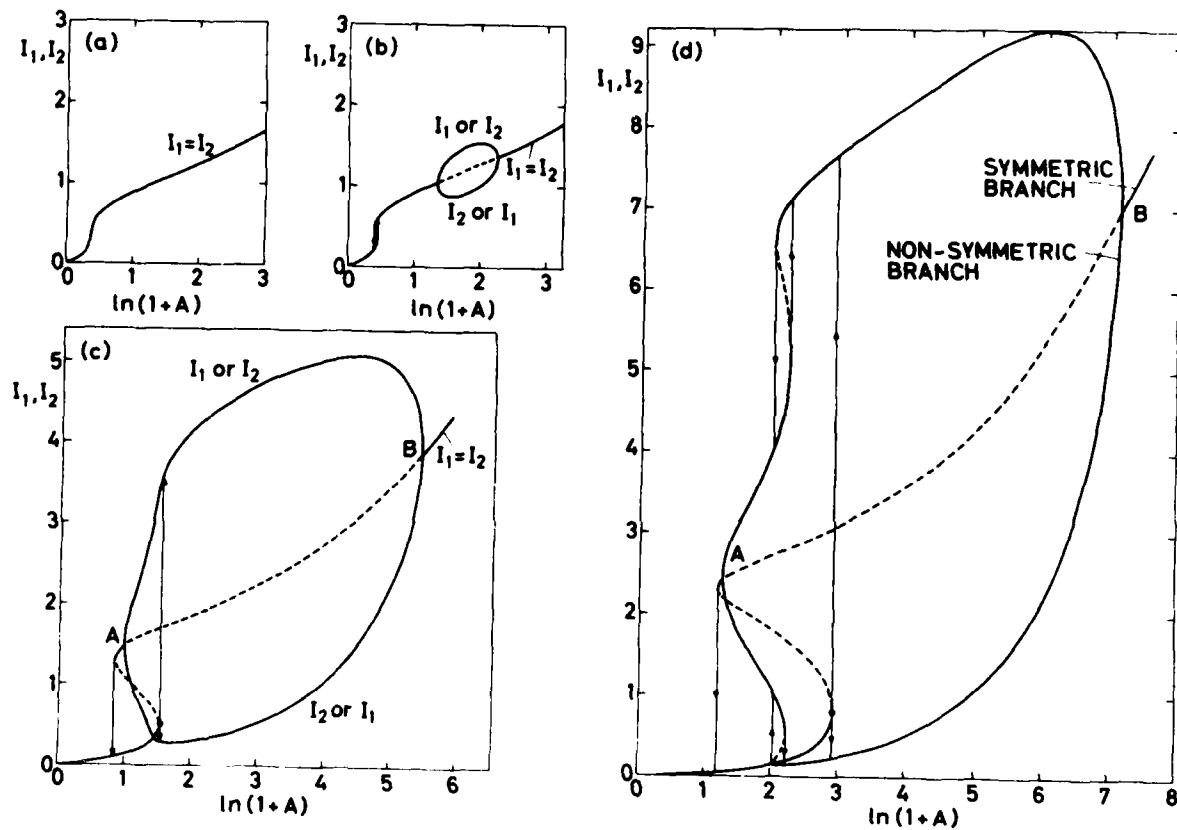


Fig. 1. Oscillation intensities  $I_1$  and  $I_2$  vs the pump intensity  $A$ . The dashed parts of the curves indicate the regions of instability. The curves labeled by  $I_1 = I_2$  give the symmetrical regime, and the points  $A'$  and  $B'$  - the crossings between symmetrical and asymmetrical regimes. The four figures are for different values of the dimensionless detuning  $\Delta$ . (a)  $\Delta = -1.65$ ; (b)  $\Delta = -1.8$ ; (c)  $\Delta = -4$ ; (d)  $\Delta = -7$ .

Vacuum-Deposited Thin-Film Interferometers  
as Bistable Devices Operating Continuously  
at Room Temperature

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Bistable operation of a nonlinear thin-film semiconductor interferometer, fabricated by vacuum deposition, was first observed in [1]. Refractive index nonlinearity required was achieved in a ZnS-spacer of submicron thickness with the input light beam intensity of about  $6 \text{ kW/cm}^2$ .

Later we also realized a bistable regime in nonlinear interferometers made of other materials, such as ZnSe,  $\text{MgF}_2$ ,  $\text{Na}_3\text{AlF}_6$ , and we think that logic response can be achieved in most thin-film interference filters described in [2].

Nonlinear thin-film interferometers as bistable devices have the advantage of operating at room temperature in the cw regime. A prolonged effect of the input light beam on ZnS-interferometers does not cause any changes resulting in a decrease in light-induced photorefraction, and this bistable element can remain in any of its stable states until the switching signal appears. In non-linear thin-film interference filters with ZnSe and  $\text{Na}_3\text{AlF}_6$  spacers, the switching of the input intensity is first followed by a short-term (a few seconds) transient after which operation of the bistable device stabilizes.

In all the thin-film interferometers studied we observed the beam profile hysteresis and an unusual form of the hysteresis loop with the negative slope of the upper branch for the on-axis zone of the input beam. The decrease



in the output intensity as the input intensity increases after the bistable device is switched on is likely to be determined by the difference between the stationary spatial profile of the light field distribution inside the interferometer and the temperature distribution profile.

Unlike thin-film filters made of ZnS, ZnSe, and  $\text{Na}_2\text{AlF}_6$ , bistable interferometers with the  $\text{MgF}_2$  spacer do not demonstrate stable operation. The "backward" hysteresis loop is characteristic for them and on achieving certain input intensity transition to the regime of regenerative pulsations is observed. Like in [3], these pulsations can be explained by the competition between processes which lead to the refractive index changes with opposite signs. In our case these processes are the light-induced photorefraction which increases  $n$ , and a slower negative change of the spacer refractive index initiated by heating.

Another advantage of the bistable devices based on evaporated thin-film interferometers is a comparatively low threshold of the bistable regime. For example, input power of 7.5 mW is sufficient to switch on the ZnSe-device with the detuning equal to a quarter of the peak width at half-transmittance. Since the focused beam has a diameter of  $50 \mu$  this light power corresponds to the input intensity of  $400 \text{ W/cm}^2$ .

The experimentally observed switch-on and -off times of our bistable interferometers are equal to 8 and  $17 \mu\text{s}$ , respectively. However, we do not consider these figures to be the principal limits. The speed of the bistable device is restricted by characteristic times of light-induced refractive index change relaxation, and a special experiment showed that these times can be as short as  $0.1\text{--}0.5 \mu\text{s}$ . In

the same experiment the induced absorption was found corresponding to the band gap of the materials used. The change of the imaginary part of the complex refractive index turned to be equal to 0.002 which corresponds to the coefficient of light-induced absorption of about  $400 \text{ cm}^{-1}$ .

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# NOTES

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**Multistability due to the helical optical  
distributed feedback in the cholesteric liquid  
crystals**

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**Summary**

It is shown that optical helical distributed feedback may cause the appearance of the multistability in transmission and reflection of the laser light by the cholesteric liquid crystal (CLC) (where helical distributed feedback laser was proposed /1/ and realized /2/) show that multistability may be realized with the intensities  $1 \text{ kW/cm}^2$  with response time  $10^{-9}$  sec.

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### Bistable Injection Lasers

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Semiconductor lasers with inhomogeneous current injection have been proposed nearly twenty years ago [1] as highly compact and efficient bistable devices. Recently, we demonstrated that a semiconductor laser with a segmented contact, as shown in Fig. 1, displays bistability without pulsations. The key to the proper design of such a bistable laser is the electrical isolation between the two segments which requires that the parasitic resistance ( $R_p$  in Fig. 2) between the two contacts be as large as possible. This can be achieved by doping the top cladding layer only slightly p-type.

When both sections of the laser are pumped equally, the light current characteristic is linear as shown in Fig. 2, curve (a). For bistable operation, the absorber section is pumped with a very small negative constant current  $I_2$  (to introduce the saturable absorption) and the light output as function of the current through the gain section displays a giant hysteresis ( Fig. 2 ,curve (b)). The amount of saturable absorption can be easily adjusted by changing the current  $I_2$  through the absorber section resulting in a different size of the hysteresis ( Fig. 2 curve (c)).

Crucial to the understanding of the characteristics of this bistable laser is a negative differential resistance (which is opto-electronic in origin) across the absorber section, as shown in Fig. 3, reminiscent of a tunnel diode. Like tunnel diodes, which can operate as bistable devices or as self-sustained oscillators depending on the bias resistor, the double contact laser can operate also in a bistable or unstable mode by choosing the proper load resistance ( $R_L$  in Fig. 2 ) at the absorber section. We have observed that a small load resistance causes the laser to pulsate and that a large load resistance causes bistability [2].

The laser has two stable states when biased within the hysteresis ( i.e.  $I_1=25\text{mA}$ ;  $I_2=-110\mu\text{A}$  in Fig.2 ), an off state ( the laser emits only spontaneous radiation ) and an on state ( the laser radiates stimulated emission ). A small positive current pulse superimposed on  $I_1$  increases the gain momentarily, thus bleaching the saturable absorption and the laser switches on. Switching off is achieved by a subsequent negative current pulse. While the switching off is usually fast, the switching on dynamics depends critically on the trigger pulse amplitude. For small trigger pulse amplitudes these switching delays are very long but they can be reduced by increasing the trigger pulse to typically 5 ns at power delay products of 100 picojoules.

We will present a simple model which explains the main features of an injection laser with inhomogeneous current injection. A set of three rate equations is used, one for the density of the carriers in the gain section, one for those in the absorbing section and one for the density of photons in the lasing mode. The calculated light current characteristic is shown in Fig. 4. It is a unique feature of semiconductor lasers that the quasi-Fermi levels of the inverted population can be accessed directly since the externally applied voltage is equal to the difference of the quasi-Fermi levels. Many interesting effects in injection lasers are a direct consequence of this fact. For a complete description, two equations relating the voltage across the absorber section and the gain section to the respective carrier densities have to be added to the three rate equations. The calculated current voltage characteristic of the absorber section shows a negative differential resistance as shown in Fig. 5.

The switching dynamics of this bistable laser shows the critical slowing down as expected from an analogy between a bistable laser (which is explicitly a nonequilibrium system) and a first order phase transition (which relies implicitly upon the assumption of thermal equilibrium). We will present a small signal interpretation of the critical slowing down.

Experimental results of this device placed in an external optical cavity are also reported. Depending on the biasing condition the device will be bistable ( $R_2 = 200\text{k}\Omega$ ) or pulsating ( $R_2 = 330\Omega$ ) as shown in Fig. 6. In the bistable mode the laser can be switched on and off by varying the amount of optical feedback and it can be used as an optical stylus to read an optical disk. In the pulsating mode ultrashort pulses through passive mode locking can be generated at a repetition rate corresponding to the round trip time in the external optical cavity.

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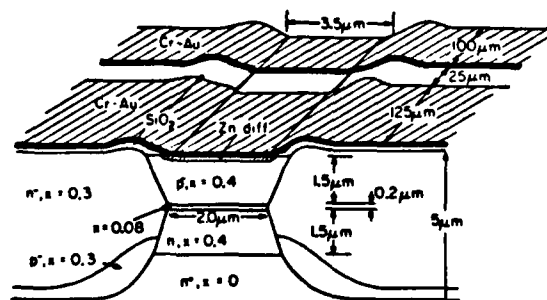


Figure 1:  $\text{Ga}_{1-x}\text{Al}_x\text{As}$  buried heterostructure laser with a segmented contact.

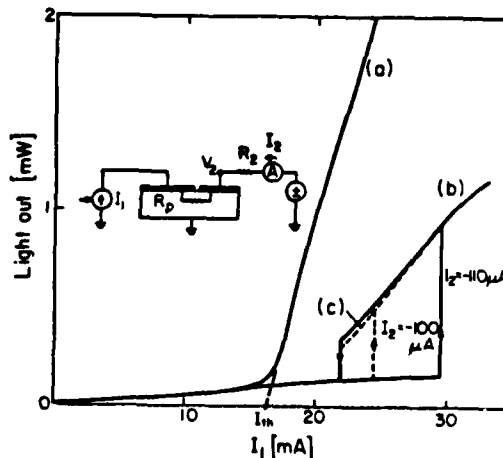


Figure 2: Measured static current light characteristic of the two segment contact laser. For homogeneous pumping: curve (a) and for inhomogeneous pumping: curve (b) and (c). Curve (b) corresponds to the case of a large saturation intensity of the saturable absorber and curve (c) to the case of a small saturation intensity.



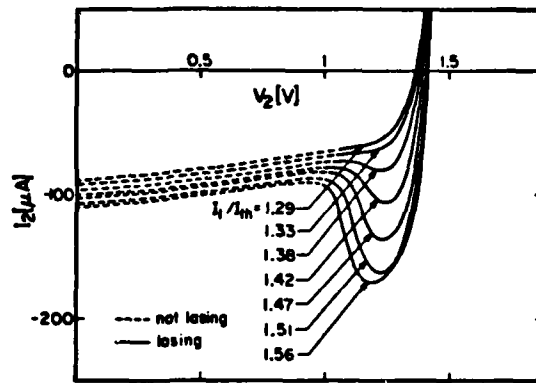


Figure 3: Current voltage characteristic of the absorber section for different pump currents in the gain section  $I_1/I_{th}$ .

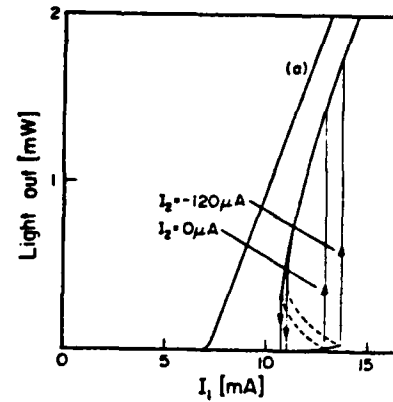


Figure 4: Calculated static light current characteristic. For homogeneous pumping: curve (a) and for inhomogeneous pumping: curve (b) and (c).

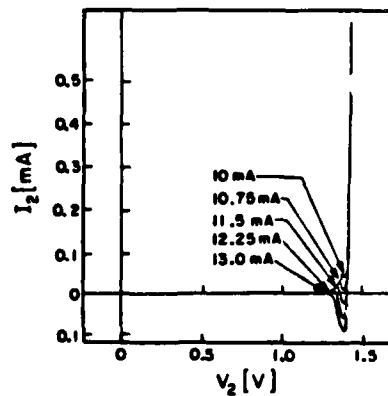


Figure 5: Calculated current voltage characteristic of the absorber section for different pump currents in the gain section  $I_1$ .

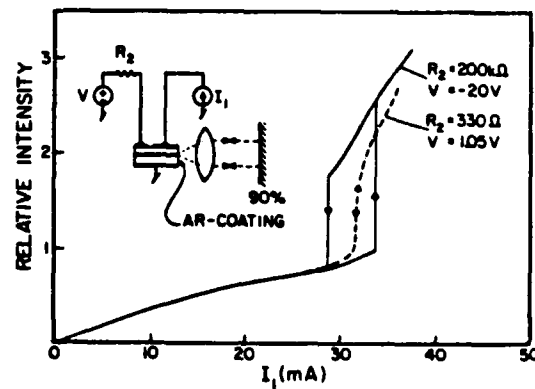


Figure 6: Light current characteristic of the device in an external optical cavity for current biasing ( $R_2 = 200k\Omega$ ,  $V = -20V$ ) and voltage biasing ( $R_2 = 330\Omega$ ,  $V = -1.05V$ ). The device is b stable for current biasing and pulsating for voltage biasing.

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As it is well known from nonlinear optics, the dielectric function  $\epsilon(\underline{Q}, \omega)$  and the resulting polariton dispersion are functions of the excitation intensity. Since in CuCl biexcitons with energy  $E_{Bi}$  may be created by the absorption of two photons provided by different light sources,  $\epsilon(\underline{Q}, \Omega)$  depends on the density of photons with frequency  $\omega_\ell$ , defined such that :

$$\hbar\Omega + \hbar\omega_\ell = E_{Bi} \quad (1)$$

Therefore, if the sample is excited by a strong laser beam with a frequency  $\omega_\ell$ , the complex dielectric function will be appreciably modified at the frequencies  $\Omega$  which fulfil equation (1) <sup>1,2</sup>. This effect has been observed recently by two photon Raman scattering in CuCl <sup>3</sup>. It is well suited to generate dispersive optical bistability. As the effect is due to virtual creation of biexcitons, it should respond within the transverse relaxation times of exciton and biexciton, which are of the order of pico-second <sup>4,5</sup>.

In our experiments, CuCl samples of the order of 30  $\mu\text{m}$  thickness are placed between two glass plates which are coated with platinum films. Their reflection coefficient is chosen to be about 90 % and gives rise to a transmission variation of 40 %. The obtained Fabry-Perot etalon is placed in a pumped helium dewar working at 1.9°K.

Concerning the exciting source, we use an excimer laser (Lambda Physik EMG 101) with XeCl as active medium to pump a dye laser working

with a diluted solution of  $\alpha$ NND in ethanol. The obtained laser beam has a narrow spectral width (0.05 meV FWHM) and a well defined time dependence. Transmission of this beam through the F.P. is detected by a fast photocell (UVHC 20) registered by a 7104 Tektronix oscilloscope using a  $\text{GH}_2$  amplifier. A part of the beam is taken as reference before entering the F.P. and, after an optical delay, detected by the same photocell. The overall time resolution amounts to 400 picoseconds.

If the photon energy of the dye laser  $\hbar\omega_\ell$  is tuned around half the biexciton energy (Eq. 1), optical bistability is clearly observed. It shows up in the deformation of the pulse transmitted through the F.P., compared to the reference pulse. The switch on and off intensities correspond to 40 and 20  $\text{MW}/\text{cm}^2$  respectively. Both commutation times are beyond our time resolution.

The effect is strongly resonant ( $\pm 3$  meV) on the biexciton energy. If the laser is detuned from the resonance, no effect could be detected. The photon energy region in which optical bistability is observed depends on the relative position of half biexciton energy with respect to a transmission maximum of the F.P. A systematic study of this point will be performed.

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Radiation Pressure Induced Optical Bistability

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Over the last few years optical bistability has been predicted and/or observed in a number of configurations involving a nonlinear medium placed inside an interferometer and irradiated by a laser field <sup>1</sup>.

In this communication, we discuss the conceptually simplest system to exhibit optical bistability. Here, no nonlinear medium is required. Rather, we take advantage of radiation pressure to achieve hysteresis in the transmission of an interferometer <sup>2</sup>.

Consider an interferometer consisting of a fixed mirror and a second mirror allowed to move freely, with some restoring force. This can be achieved for instance by hanging it with a thin wire, so that its motion is that of a pendulum. In the presence of a radiation field inside the resonator, its equation of motion is

$$\ddot{\zeta} = -\omega_0^2 \zeta - 2\gamma \dot{\zeta} + \frac{\alpha^2}{m} |E(t)|^2, \quad (1)$$

where the last term is the radiation pressure force.

Here,  $\zeta$  is the displacement of the mirror from equilibrium,  $\omega_0$  its eigenfrequency,  $\gamma$  its damping constant,  $m$  its mass, and  $E(t)$  the electric field inside the resonator.

$$\alpha^2 = \frac{R}{2} \epsilon_0 S, \quad (2)$$

where  $R$  is the reflection coefficient of the mirror,  $\epsilon_0$  the dielectric constant of vacuum, and  $S$  the irradiated area. In deriving Eq. (1), we have assumed that the mirror exhibits small displacements only.

The equation of motion for the transmitted field is, as usual <sup>3</sup>

$$\dot{E}_T = \kappa E_I - (\kappa + i\Delta) E_T, \quad (3)$$

where  $\kappa = cT/L$  is the linewidth of the resonator,  $E_T = E\sqrt{T}$  is the transmitted field, and

$$\Delta = R(\omega_c - \omega) = R\left[\omega_c^0 / \left(1 + \frac{2\zeta}{L_0}\right) - \omega\right]. \quad (4)$$

Here,  $L_0$  is the rest length of the resonator and  $\omega_c^0$  the corresponding frequency, while  $\omega_c$  is the corresponding eigenfrequency for a length  $L = L_0 + 2\zeta$ .

At steady-state, Eqns. (1) and (3) yield

$$Y = X \left[ 1 + \left( \frac{R}{\kappa_0} (\omega_c - \omega) - \frac{R}{cT} \left( \frac{2\omega}{m} \right) \frac{\chi^2}{\omega_c^2} X \right)^2 \right], \quad (5)$$

where  $Y = |E_I|^2$  and  $X = |E_T|^2$ , and  $\kappa_0 = cT/L_0$ .

Thus, we obtain a cubic relationship between incident and transmitted field, and optical bistability. Using very light mirrors (1-100 mg), it can be shown easily that bistability can be reached for very reasonable powers of 1-200 mWatt. We note that this bistability is in a sense very similar to dispersive bistability with a Kerr medium, except that here, the effective change in optical length is replaced by a real change of cavity length. This system opens a number of applications, in particular in sound waves to optical signal transducers, very sensitive scales, etc...

In the presentation, a detailed time-dependent analysis of this system, its experimental verification, and possible applications, will be discussed.

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2. A preliminary presentation of this work has been given at the Spring Meeting of the German Physical Society, Regensburg, 1983
3. See for instance J.D. Cresser and P. Meystre, in Ref. 1

# NOTES



NOTES

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